PERTURBED ROTATIONAL MOTION OF A RIGID BODY

Thesis

Submitted In Partial Fulfillment of The Requirements of The Degree of Master of Teacher Preparation In Science (Applide Mathematics)

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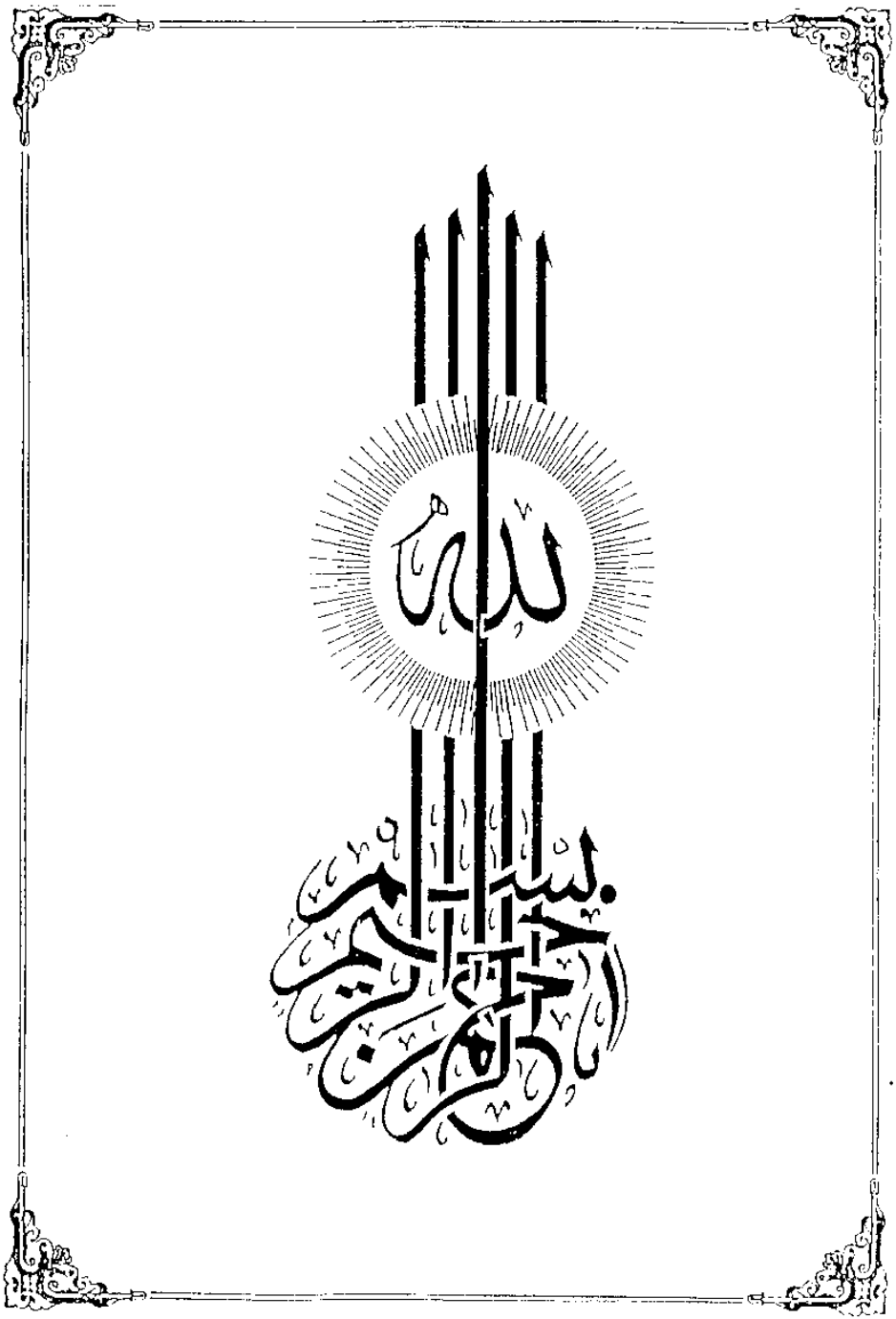

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SUMMARY

Summary

General characteristics of thesis

1- Actuality subject :

In the present work, the perturbed rotational motions of a rigid body relative to fixed point under the action moments of force due to different physical nature are studied. We considered the motion of a rigid body, close to the case of Lagrange at presence of small perturbed moments, arising from the following influences:

a) Resistance media.

b) Constant moments in the directions of the principal axes of inertia of the body.

c) Linear dissipative moments in the directions of the principal axes of inertia of the body.

d) Mass distribution, close to the Lagrange case.

These problems arise in case of studying the motion of rigid bodies relative to the center of mass, in questions of orientation and stability of cosmic apparatus, in the gyroscopic dynamic. The differential equations of these systems are nonlinear and the study of them may be difficult. Approximate methods are powerful means for studying the problem of dynamic rigid body. For analysis the nonlinear system of differential equations of motion, we used the average method .

2- The aim of work:

The aim of work is confined in studying evolution of the perturbed rotational motions of rigid body close to the cases of Lagrange under the action of moment of force due to different physical nature in case of variable restoring moment.

3- Scientific novelty:

The getten results consist of the following:

(a) Studying the perturbed rotational motion of a rigid body close to reguler precession in the case of Lagrange when the restoring moment depends on the nutation angle.

(b) Studying the perturbed rotational motion of a rigid body close to regular precession in the case of lagrange in the presence of electro -magnetic field.

4- Practical importance:

The thesis is enclosed in that qualitative and quantitative analysis of motion of rigid body under the action of series of perturbations, which we meet in practice of dynamic satellite and gyroscopic.

5- Contents of thesis:

The thesis consists of an introduction and three chapters:

The introduction is giving a survey on the works related to the subject of the thesis, also disscused some questions related to similer works.

CHAPTER II, Deals with the study of the perturbed rotational motion of a symmetrical rigid body that is close to regular

precession in the Lagrange case acted upon by the sum of two perturbed moments, one of them is constant relative to the body axes and the other is linear dissipative and in the case that the restoring moment is depending on the nutation angle θ, which is introduced by considering the rigid body rotates under the action of gravitational force and elastic force of a spring which is connected with it. The equations of motion are formulated and the following initial conditions were taken into account. These initial conditions meant that; the direction of the angular velocity of the body is close to the axis of dynamic symmetry, the angular velocity is large, two projections of the vector of the perturbing moment onto the principal axes of inertia of the body are small as compared to the restoring moment, while the third is of the same order of magnitude as this moment. These assumptions allowed us to introduce a small parameter $\in$ which causes the perturbed motions.

Given a theoretical description in a general form in the cases of resonance and nonresonance techniques for the average method which is employed for solution the system of equations of motions is the first approximation. The angles of Euler and the components of the angular velocity are expressed them and a comment for each one is given. The obtained solutions have one error of order ε and in an interval of time variation of order ε^{-1}.

CHAPTER III; is devoted to studying the problem of perturbed rotational motion of a rigid body with mass distribution near to the

Lagrange case acted upon by the last mentioned restoring moment. The mass distribution is taken, such as the ellipsoid of inertia relative to a point 0 is close to being an ellipsoid of revolution, so that its principal moments of inertia have the form $A=A^{\circ}(1+\varepsilon\delta_1)$, $B=A^{\circ}(1+\varepsilon\delta_2)$ and $A^{\circ}\neq C$, where δ_1, δ_2 are dimensionless constants of order unity and represent parameters of perturbed moment and A° is the charecteristic value of moment of inertia. The equations of motion are formulated and the perturbed moments are intorduced. Here the initial conditions as mentioned before in chapter I, except that each one of the components of perturbed moments is less than the value of restoring moment. Given a general description for the average method in the cases of first and second approximations, this method is employed in solving the system of differential equations of motion. The Euler's angles θ, ψ are expressed in the second approximation. The obtained solutions have an error of order ε and in an interval of time variation of order ε^{-1}.

CHAPTER IV is devoted to studying the perturbed rotational motions of a symmetrical rigid body that are close to regular precession in the Lagrange case acted upon by moments of force due to different physical nature in the case that the restoring moment is depending on the Euler's angles and the components of angular velocity. Such restoring moment is introduced by making the rotation of the rigid body in an electro-magnetic field. The equations of motion are formulated with the initial conditions