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**ELECTROHYDRODYNAMIC
STABILITY OF A FLUID LAYER
BOUNDED BY TWO CYLINDRICAL
INTERFACES UNDER A PERIODIC
ELECTRIC FIELD**

A THESIS

*Submitted in Partial Fulfilment of the Requirements of the Award
of the Master of Science Degree*

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**DEDICATED TO
MY DAUGHTER HEBA**

NOTE

This thesis is submitted to Ain Shams University in partial fulfilment of the requirements of the Master of Science Degree in Applied Mathematics.

Besides the research work in this thesis, the candidate has attended six postgraduate courses within the year (1981 - 1982) including the following topics:

- (1) Theory of Stability
- (2) Fluid Mechanics
- (3) Electromagnetic theory and Magnetohydrodynamic
- (4) Theory of Elasticity
- (5) Classical Mechanics
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SUMMARY

SUMMARY

This thesis is mainly concerned with the electro-hydrodynamic stability of two interfaces separating three fluids. The fluids are dielectrics and are stressed by a periodic electric field.

Our interest is to examine the effect of a periodic field on the stability of two cylindrical interfaces.

The following problems are investigated:

- (1) The EHD stability of a fluid layer bounded by two cylindrical interfaces and acted upon by an axial periodic field.
- (2) The EHD stability of a hollow jet under the influence of a constant radial field.
- (3) The EHD stability of a hollow jet stressed by a radial periodic electric field.

In Chapter one we introduce the subject of electrohydrodynamics. The equations governing the motion and the boundary conditions which are satisfied by the solutions are derived. The basic concepts of the hydrodynamic stabilities are explained and the perturbation equations are derived. We introduce the previous works and the fundamental aspects on the theory of jets stability and

the parametric excitation of systems. The techniques followed in this respect such as the method of multiple scales are introduced. On the other hand the properties of Mathieu functions and their characteristic curves are presented.

Chapter two deals with the electrohydrodynamic stability of two cylindrical interfaces separating three different inviscid dielectric fluids and stressed by a tangential periodic electric field. We solve the equations of motion governing a small disturbance to which the system is subjected. Application of the boundary conditions lead to two simultaneous ordinary differential equations of Mathieu type which are solved by the method of multiple scale perturbations; for a small amplitude of an electric field, the stability conditions are discussed. It is found that the constant tangential field has a stabilising effect while the tangential periodic field has stabilising influence except at resonance points. Graphs are drawn to illustrate the resonance regions in a parameter space. It is also found that the thickness of the jet plays a role in the stability criteria. The frequency of the modulated field can be used to control the position of the resonance regions. The special cases of large modulation and small modulation are also examined. It is found that for large

modulation, the field exhibits more destabilising influence.

Chapter three is concerned with the electrohydrodynamic stability of a hollow jet acted upon by a radial field, as in the previous Chapter, we obtain two coupled Mathieu equations. As a limiting case, we discuss the stability of the system under the absence of the periodicity of the electric field. We obtain the necessary and sufficient conditions for stability. These conditions are treated theoretically, numerically and graphically.

It is found that the constant radial field has a destabilising influence and the system can not be stabilised for $x < 1$ ($x = ka$, k is the wave number and a is the inner radius of the jet). For $x > 1$ the system can be stable provided that the electrode potential should not exceed a critical value. The reduction of the thickness of the jet plays a stabilising role in contrast with the case of an axial field. The increase of the inner radius of the jet improves the stability conditions.

Chapter four deals with the study of the

electrohydrodynamic stability of a hollow jet under a periodic radial electric field. The two coupled Mathieu equations, obtained in Chapter II, are solved by the method of multiple scale perturbations. For small electrode potential, stability conditions are discussed. Resonance regions are observed with regions which were considered stable under a normal field. The periodicity did show stabilising effect on the regions which were stable under a constant field.

CHAPTER (I)
INTRODUCTION

CHAPTER I

INTRODUCTION

(1.1) FOUNDATION OF ELECTROHYDRODYNAMICS .

Electrohydrodynamics include that part of fluid mechanics concerned with electrical force effects. It can alternatively be considered as that part of electrodynamics which is concerned with the influences of the moving media. Many of the interesting problems in electrohydrodynamics involve both an effect of the fluid motion on the fields, and conversely, an influence of the fields upon the motion. The term "electrohydrodynamics" is relatively new, but the area it treats is not new. The relevant literature is as old as that for the subject of electricity itself.

Applications of electrohydrodynamics include pumping and levitation of liquids and gases, extraction of containments from gases such as smoke [38], mixing of liquids, orientation of liquids [4] in near-zero gravity environments, augmentation of heat transfer [7] , and property measurements in fluid systems. Electrohydrodynamic interactions also occure in meteorology [15], in which charge distribution in the atmosphere is important, and in surface physics in which the distribution of charges