ON THE MULTIOBJECTIVE PROGRAMMING PROBLEM

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CONTENTS

INTRODUCTION

The vast majority of mathematical programming models from Operations research involve problems with one objective function, however, many of the optimization problems which have been addressed by classical single objective models are in fact a multiobjective in nature. Thus, there is some value in having the capability of considering more than one objective simultaneously and explicitly in several kinds of models. Multi objective mathematical programming is one way of considering multiple objectives explicitly and simultaneously in a mathematical programming framework. Much of the research in this important area has occured since 1970, however, the work of the 70's and early 80's has its root in earlier times, for example, in 1951, koopmans gave proofs of necessary and sufficient conditions for efficiency and Kuhn and Tucker [18] formulated the vector maximization problem.

There are several reasons for the increasing interest in multiple objective programs. The most important reason is the recognition that most decision problems in economics are inherently multiple objective. Examples of such problems are project management problems, inventory planning problems, location problems, capacity expansion problems, etc. Another reason is the enormous improvement over the last 15 years in the speed , storage, and flexibility of computing facilities. Algorithms for solving multiple objective mathematical programs typically require much more storage and CPU time than algorithms of similar single objective models, i.e ,differing only in the number of objectives considered.

The efficient solution to the multiple objective program is considered as a technical interpretation of the multiple objective solution; it is the most preferred solution for the multiple objective decision problem. In order to help the decision maker identifying a most preferred solution to a linear multiple objective programming problem, the set of all efficient solutions may be presented to the decision maker. Thus the characterization and determination of the set of all efficient solutions have become one of the main targets in the last decade.

In this thesis we present an algorithm for finding all efficient extreme points of the multi objective linear programming problem (MOLPP). Algorithms for enumerating all efficient solutions to the MOLPP have been developed by many authors. Among others, the algorithms for the enumeration of the set of all efficient extreme points have been developed by Charnes and Cooper [6] , Yu-Zeleny [24] , Zeleny [25], and Gal who [13] makes use of the multiparametric linear programming. Isermann [18] proposed an algorithm for finding the set of all efficient extreme points of MOLPP, which can be regarded as a dual method to parametric programming.

Virtually all of the techniques used to solve MOLPP are extensions of the techniques employed for solving single objective linear programs. In general, algorithms which solve MOLPP'S can be divided into two categories:-

(i) those which concentrate on finding all efficient extreme points, and

(ii) those which concentrate on finding efficient points regardless of whether those points are extreme or not.

Algorithms in the first category generally consist of two phases. Phase I consists of finding an initial efficient extreme point. Phase II involves finding all remaining efficient extreme points. Phase I is relatively easy to execute, as it typically only requires procedures from classical linear programming theory. Phase II is the part of the process that causes difficulty and inwhich the various algorithms differ in their approaches.

Several efforts have been spent on developing procedures for fully describing a convex polyhedral set, among others, Motzkin, Balinski, Manas and Nedoma, Mattesis and Raiffa have devised computational methods for finding all extreme points of a convex polyhedron, each with a different approah.

The main purpose of this study is to develop a method for generating all efficient extreme points of the MOLPP. In chapter IV, we propose a method for finding all efficient extreme points of the MOLPP. The method has some new features and is based on Manas and Nedoma's method and the revised simplex theory .

The thesis is divided into five chapters. In chapter I, we present the basic concepts and theorems concerning the linear programming problem and the MOLPP. Chapter II is a review of three methods that find all extreme points of a convex polyhedron. Chapter III is mainly a review of three known methods for finding the set of all efficient extreme points. In chapter IV we introduce an algorithm with new features for finding all efficient extreme points of the MOLPP. The algarithm is based on the multicriteria simplex method, but using the product form of the revised simplex theory, for the enumeration of the set of all efficient extreme points.

The"book-keeping"for the proposed algorithm has been designed so as to minimize the necessary data stored. It requires only four numbers, instead of a list of the indices of the all basic variables, for each extreme point. In chapter V, we give a new computer package program for the proposed algorithm. The program has been written in FORTRAN IV language and is self documented, i.e, contains all needed comments. In the appendix, we present the computational results of the examples and test problem used to test the program.

CHAPTER I

BASIC CONCEPTS AND THEOREMS

Introduction:

This chapter deals with the basic definitions and theorems of the linear programming problem and then the corresponding concepts and theorems in the case of the multiple objective linear programming problem. Section 1 involves mainly the statement of the linear programming and the definition of the convex sets, also we mention some basic theorems of the linear programming. In section 2 we deal with the simplex method as a technique for solving linear programming problems. The product form of the revised simplex method is presented in section 3. Section 4 deals with the duality in linear programming problems. Short description of the parametric linear programming is given in section 5. In section 6 all the preceeding concepts that correspond to the multiple objective linear programming problems are presented.

1.1. Preliminaries:

1.1.1: The Linear programming problem (LPP): [20]

The LPP can be enunciated as follows;

Let $x \in E_n$; $f(x)$ and $g(x)$ be linear functions defined as:

$$f(x) = \sum_{j=1}^{n} c_j x_j$$

$$g_i(x) = \sum_{j=1}^{n} a_{ij} x_j - b_i, \quad 1 \leq i \leq m,$$

$$c_j, \; a_{ij} \text{ and } b_i \in R, \quad 1 \leq j \leq n,$$

E_n is the Euclidean space of dimension n.

To find the n-vector x_o such that:

$$f(x_o) \geq f(x) \quad \text{for all points } x$$

satisfying the constraints:

$$g_i(x) = 0 \quad 1 \leq i \leq m,$$

and

$$x \geq 0.$$

The Lpp can be stated in the matrix form as follows:

Maximize	$f(x) = cx$	(1.1)
Subject to	$Ax = b,$	(1.2)
and	$x \geq 0,$	(1.3)

where c is an n th row vector, x is n th column vector and b is mth vector, A is an (mxn) matrix, and $m < n$.

Expression (1.1) is the objective function to be maximized, (1.2) are the constraints, and (1.3) the non negativity conditions. Equations(1.3) are also constraints but because of their simplicity are treated seperately from (1.2). The coefficients c_j are usually called the cost coefficients.

Any LPP can be transformed to the forms (1.1),(1.2) and (1.3) as shown in the sequel.

If f (x) is to be minimized then we may put $h(x) = -f(x)$ which is to be maximized. If a constraint is an inequality then it can be converted to an equation by introducing an extra variable with a constraint imposed on it, i.e ,

$$g_i\ (x) \leqslant (\geqslant)\ 0$$

if and only if

$$g_i\ (x) + x_i\ (-x_i) = 0,$$

where

$$x_i \geqslant 0, \quad 1 \leqslant i \leqslant m.$$

The variables x_i so introduced are called slack variables.

If a variable x_j is unconstrained, i.e , it may vary from $-\infty$ to ∞ , then we may put x_j by two other variables x_j^+ and x_j^- such that:

$$x_j = x_j^+ - x_j^- ,$$

$$x_j^+ \geqslant 0,$$

$$x_j^- \geqslant 0.$$

A solution to (1.2) and (1.3) is called a feasible solution. We shall denote by X the set of all feasible solutions. It is possible that there may exist no solution to (1.2), in that case X is an empty set.

The matrix equation (1.2) consists of m equations in n unknowns, we shall assume that the equations are linearly independent. If any of n-m variables x_j are given the values zero, the remaining system of m equations in m unknowns may have a solution, this solution along with the assumed zeros is a solution to (1.2), it is called a basic solution. The m variables remaining in the system after (n-m) variables have been put equal to zero are called the basic variables or simply the basis, the rest of the variables are called the non basic variables. Since the unique solution of the m equations in m variables may also contain zeros, then a solution containing a number of zeros more than (n-m) is called a degenerate basic solution.

Equation (1.2) may be written as:

$$\sum_{j=1}^{n} a_{ij}\, x_j = b_i \quad , \ i = 1, \ \ldots\ldots, \ m,$$