STUDIES ON THE COMPLEX PERMITTIVITIES OF SOME DIELECTRIC MATERIALS AT MICROWAVE FREQUENCIES

THESIS

Submitted for the Degree of

DOCTOR OF PHILOSOPHY

In Physics

By

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B.Sc., M.Sc. (Physics)

To

The Faculty of Science

Ain Shams University

Cairo - Egypt

1986

ACKNOWLEDGMENT

This work is a result of the sincere cooperation of the scientific channel between **Prof. Dr. H. Severin** - Director of the Institute of High and Ultra High Frequency Techniques, RUHR University-Bochum, West Germay and **Prof. Dr. M.A. El Sharkawy**, Prof. of Electronics, Physics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.

The author wishes to express his sincere gratitude to **Prof. Dr. A.A. Mohamed**, Head of the Physics Department, Faculty of Science, Ain Shams University.

The author is specially indebted to **Prof. Dr. H. Severin** for his great help and continuous supervision and valuable suggestions during the research period in Bochum.

The author is specially indebted to **Prof. Dr. M.A. El Sharkawy** for his continuous supervision and valuable suggestions during the research period in Cairo.

Finally, the author would like to express his deep appreciation to **Mr. Woiska**, Chief of the mechanical workshop and the members of the workshop - Institute of High and Ultra High Frequency Techniques, RUHR University - Bochum, West Germany for their help in the machining of the material samples and cavities.

بسم الله الرحمن الرحيم

CONTENTS

Page

LIST OF SYMBOLS

Q	:	Quality factor of the cavity.
ω	:	Angular frequency.
ε_r	:	Relative dielectric constant = $\varepsilon/\varepsilon_o$.
ε_o, μ_o	:	Permittivity and permeability of free space.
$\vec{E}_x$, $\vec{E}_y$ & $\vec{E}_z$	:	Electric field component along x, y & z directions.
$\vec{H}_x$, $\vec{H}_y$ & $\vec{H}_z$	:	Magnetic field component along x, y & z directions.
a, b	:	Cross sectional dimensions of the rectangular cavity a = 22.86 mm, b = 10.16 mm.
d	:	Thickness of the dielectric slab.
β	:	Phase constant.
f_o	:	Resonant frequency of the empty cavity.
f	:	Resonant frequency of the loaded cavity.
ε'_r	:	Real part of the dielectric constant.
ε''_r	:	Loss factor (imaginary part of the dielectric constant).
K_x, K'_x	:	Transverse propagation constants in the x-direction in the air and dielectric regions inside the cavity.
K_y, K'_y	:	Transverse propagation constants in the y-direction in the air and dielectric regions inside the cavity.

INTRODUCTION

INTRODUCTION

The determination of the complex permittivities of dielectric materials are found to be of a great interest from many authors either by using cavity perturbation method or the exact solution of the dielectric loaded waveguide.

Cavity perturbation method is considered as an approximative technique which leads to acceptable results if the dielectric sample in the cavity is very small compared with the wavelength in the material. It can be highly accurate in determining dielectric constants and small loss tangents (tan δ_{ε}). The measurements can be performed for dielectric samples of various shapes as spheres, rods, discs, slabs,... etc.

For developing microwave components such as microwave filters, phase changers and matching transformers, rectangular waveguides containing a dielectric slab in the E- or H-plane are of interest and have been treated exactly. From the characteristic equations of the various structures the dielectric constant can be evaluated, if the resonance frequency of the rectangular dielectric loaded cavity is measured. In spite of the computation difficulties with this method, it leads to a very high accuracy in the dielectric constant.

This thesis is a study of four different structures of a rectangular cavity with a dielectric slab placed at the centre or at one side of the cavity either parallel to the

E-plane or H-plane. It is the aim to find out the accuracy limits for the application of the cavity perturbation method by comparing the results of the exact solution with those of the cavity perturbation method. This has been done for various values of the dielectric constant.

Additional measurements for the same cavity structures have been performed at X-band frequencies in order to find the measuring accuracy of resonator method by comparing measured and calculated values of relative frequency shift.

The work has been extended to the cavity of square cross section. In case of very thin samples degenerate modes can be used for measuring the dielectric constant, and disassembling of the cavity during the measuring process can be avoided. The accuracy limitation of the perturbation method for this case has also been found out, and measurements for the dielectric loaded cavity have been performed. The measured and calculated results have been compared in order to find again the measuring accuracy.

CHAPTER (I)

EXACT SOLUTION FOR A RECTANGULAR CAVITY LOADED WITH A LOSSLESS DIELECTRIC SLAB

In this chapter, the exact solution for a rectangular cavity loaded with a dielectric slab is presented. The dielectric slab is placed at the centre or at one side of the cavity either parallel or perpendicular to the E-plane. The transcendental equation for each structure is derived and formulated. The normalized frequency shifts are determined taking in consideration the approximations due to the relative ratio between the slab thickness and cavity dimensions.

I-1) Case (1) : Dielectric Slab Placed at the Centre of a rectangular cavity parallel to the E-plane (H_{10n}-mode).

Fig. (I-1a) represents a cross sectional view for a dielectric slab placed at the centre of a rectangular cavity parallel to the E-plane. The exact solution can be obtained by considering the propagating wave as H_{10n}-mode with E_y, H_x & H_z field components and no variation in the fields along the y-direction, therefore $\frac{\partial}{\partial y} = 0$. Since the slab is non magnetic, H_z and H_x are unaltered. Thus the solution must have $H_y = E_x = E_z = 0$ and no variation along the y-direction. The wave equations for this structure are given by [1]

$$(\nabla_t^2 + K_x)\, H = 0 \qquad \text{in the air}$$

$$(\nabla_t^2 + K_x')\, H_z = 0 \qquad \text{in the dielectric}$$

where

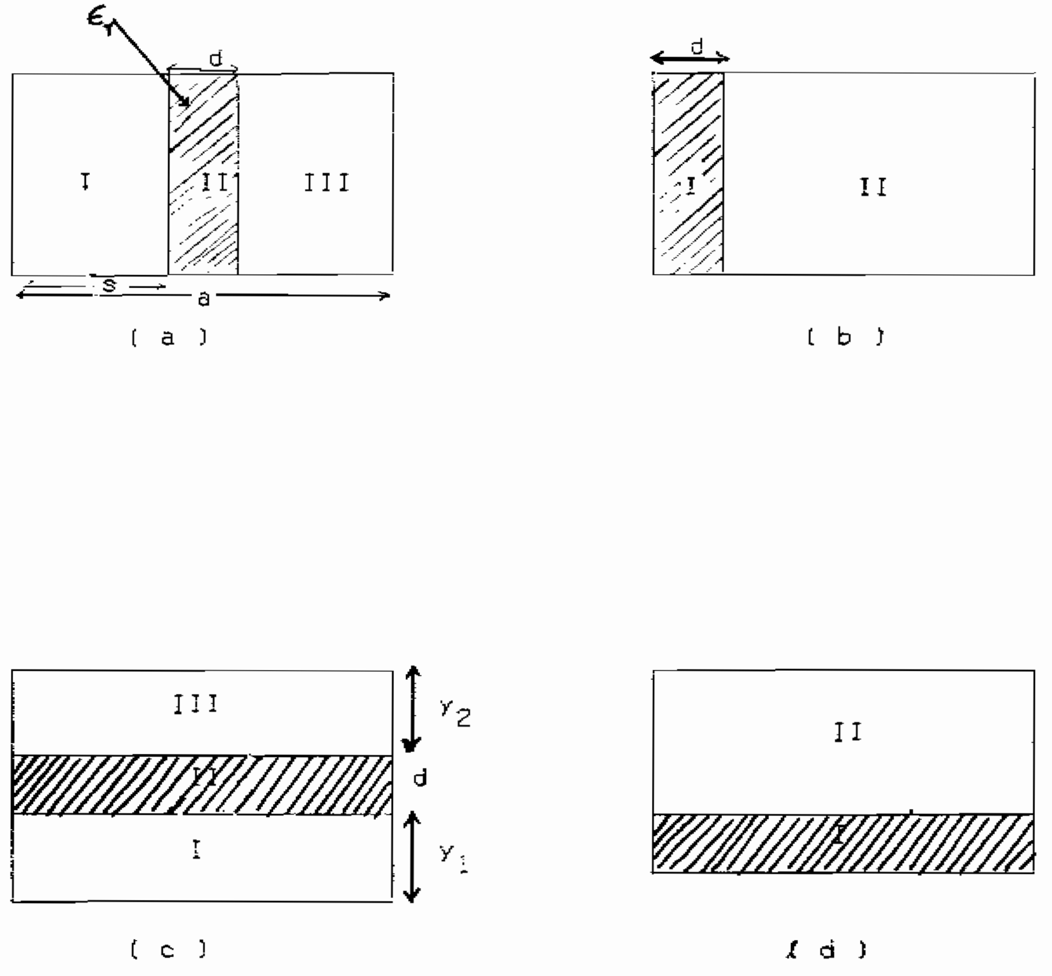

Fig. (I-1) Cross sectional view of the dielectric slab loaded rectangular cavity.

$$K_x^2 = \omega^2 \varepsilon_o \mu_o - \beta^2$$
$$(I-1)$$
$$K_x'^2 = \omega^2 \varepsilon_o \xi_o \varepsilon_r - \beta^2$$

and β is the phase constant.

In the three regions I, II, III the solutions can be written in the forms

$$H_z = A \cos (K_x x) \qquad \text{in I}$$
$$= B \cos (K_x' x) + C \sin (K_x' x) \qquad \text{in II}$$
$$= D \cos (K_x (x-a)) \qquad \text{in III}$$

where A, B, C & D are constants. The other field components are given from the relations

$$K^2 H_x = - j\beta \frac{\partial H_z}{\partial x}$$

$$K^2 E_y = j \omega \mu_o \frac{\partial H_z}{\partial x}$$

where K is either K_x or K_x' depending on the region. In all field components the factor exp $(j\omega t)$ and exp $(-j\beta z)$ are omitted for simplicity. From the boundary conditions, H_z and E_y must be continuous at the air-dielectric interfaces i.e. at $x = s$ & $x = t$, $t = s+d$. Applying these boundary conditions four equations in A, B, C & D are obtained as

$$\begin{bmatrix} K_x' \sin(k_x s) & -K_x \sin(K_x' s) & K_x \cos(K_x' s) & 0 \\ 0 & K_x \sin(K_x' t) & -K_x \cos(K_x' t) & K_x' \sin(K_x s) \\ \cos(K_x s) & -\cos(K_x' s) & -\sin(K_x' s) & 0 \\ 0 & \cos(K_x' t) & \sin(K_x' t) & -\cos(K_x s) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Equating the determinant of these equations to zero, the characteristic equation of the structure is obtained as

$$K_x^2 \cos^2(K_x s) \sin(K_x' d) + 2K_x' K_x \sin(K_x s) \cos(K_x s) \cos(K_x' d)$$

$$- K_x'^2 \sin^2(K_x s) \sin(K_x' d) = 0$$

In terms of the half angles and dividing by $\sin^2(K_x s) \cos^2(K_x' \frac{d}{2})$ the equation can be simplified to

$$K_x \cot(K_x s) = K_x' \tan(K_x' d/2) \qquad \text{(I-2)}$$

This equation represents the transcendental equation of the structure which is in agreement with R. Collin [2] and P.H. Vartanian [3].

For very small slab thickness ($d \to 0$), equation (I-2) yields to an approximated equation as

When $d \to 0$, $\tan(K_x' d/2) \to K_x' d/2$

& $\cot(K_x s) \to -\delta K_x a/2 + K_x d/2$

where $\delta K_x = K_x - K_{xo}$, $K_{xo} = \pi/a$.

Then the transcendental equation (I-2) can be written as

$$K_x' \, d/2 = -K_x \, \delta K_x \, a/2 + K_x^2 \, d/2$$

$$(K_x'^2 - K_x^2) \, d/2 = -K_x \delta K_x$$

Using equation (I-1)

$$(\varepsilon_r - 1)\omega^2 \varepsilon_o \mu_o \, d/2 = -\omega^2 \varepsilon_o \mu_o \delta \omega / \omega$$

$$\therefore \qquad \frac{\delta\omega}{\omega} = -(\varepsilon_r - 1) \, d/a \qquad \text{(I-3)}$$

When the slab thickness is very large $(d \to a)$, the transcendental equation (I-2) can also be simplified with some approximation as

$$K_x \cot (K_x s) = K_x' \tan (K_x' \, d/2)$$

$$\frac{K_x}{K_x s + K_x^3 \frac{s^3}{3} + \ldots} = \frac{K_x'(\tan (K_x' \frac{a}{2}) - \tan (K_x' s))}{1 + \tan (K_x' \frac{a}{2}) \tan (K_x' s)}$$

$$\frac{K_x}{K_x s + K_x^3 \frac{s^3}{3}} \simeq \frac{K_x' (1 - \tan (K_x' s) \cot (K_x' \frac{a}{2}))}{\cot (K_x' \frac{a}{2}) + \tan(K_x' s)}$$

$$\cot (K_x' \, a/2) = \cot (\pi/2 + \delta K_x' \, a/2) \simeq -\delta K_x' \, a/2$$

and since s is small, then

$$\tan (K_x' s) \cot (K_x' \, a/2) \ll 1$$

$$K_x(-\delta K_x' \, a/2 + K_x' s + K_x'^3 s^3/3) = K_x' (K_x^3 s^3/3) + K_x s$$

$$(K_x'^2 - K_x^2) \, s^3/3 = (\delta K_x'/K_x') . \, a/2$$