



ON THE RELIABILITY FOR REPAIRABLE SYSTEMS AND COST ANALYSIS

THESIS

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PREFACE

The interest in reliability theory currently exhibited by mathematicians, engineers, economists and industrial engineers and managers has stimulated the publication of many papers. This thesis deals with analytical and numerical methods as well as the necessary computer programs and graphical representation for studying the reliability for repairable systems .

The study deals with statistical analysis of the reliability of different repairable redundant systems. The analysis incorporates different systems that have sound applications in practical life .The system considered may be consisted of similar or dissimilar redundant units with either cold or warm standby. The systems may have different types of operation regime (repair only or repair ,preventive maintenance and inspection). The mean time to system failure (MTSF) , pointwise and steady state availability have been calculated . Probability that the repairman is busy , expected number of visits by the repairman and the profit earned by the system have been also obtained using the results of regenerative process.

The first chapter investigates the stochastic behaviour of a system consisting of two dissimilar units with common

cause failure, preventive maintenance and two types of repair (regular and expert). The system goes for preventive maintenance at random epochs, when both units are in normal operation. The regular repairman is always available to repair the failed unit. If the regular repairman is not able to do the repair, the expert repairman is called for repair. The system fails either one unit by another or simultaneously due to common cause failure. The random variable concerning failure, repair and maintenance times have general distribution. The MTSF, steady state availability, probability that the repairman is busy, expected number of visits by the expert repairman and the profit earned by the system are obtained. Three special cases were studied. The first case concerned with identical units, in the second case the failure, repair and maintenance times are exponentially distributed, the third case concerned with a single regular repair. The results in [55] have been derived as special case from the results of this chapter. Computer programs to calculate the MTSF and the profit of the two cases are developed from the second and third special cases, the first case with two types of repair (regular/expert) and the other with one type of repair (regular). From comparison, it could be concluded that the system in the first case is more available than the second (see Appendices I.1 and II.1).

The second chapter deals with the probabilistic behaviour of a system consisting of two dissimilar units

deteriorating standby (DS) with inspection. Initially one unit is operative and other (DS). The (DS) unit can fail during inspection or at the time it is needed in case of failure of operative unit. When the operative unit fails during inspection of (DS) unit, the system immediately switch on the standby into operation and stop the inspection. The random variables concerning failure, repair, inspection and inter-inspection times are assumed to be arbitrarily general distributions. The MTSF and steady state availability for this system have been obtained . Three special cases were studied , the first case concerned with similar units , in the second case the two units are similar and the failure times are exponentially distributed , while in the third case the two units are dissimilar with exponential distributions . The results given in [5] have been derived as special case from the results of this chapter . Computer program to calculate the MTSF for the third case is also given (see Appendix I.2). The study performed in this chapter has been accepted for publication in "Journal of Microelectronics and Reliability", England, 1992.

The third chapter considers two systems for two dissimilar units cold standby redundant system with different types of repair and perfect or imperfect switching . Each unit works in three different modes (normal, partial failure and total failure). In the first system, the switch is perfect to switch on the standby unit in case of total failure of the

operative unit. In the second system, the system has an imperfect switch and pre-emptive repair policy. The repair time of the switch in the second system is arbitrary with general distribution. In both systems, the failure and repair times are arbitrary with different distribution. The MTSF, steady state availability, probability that the repairman is busy and the expected profit earned by the system are obtained for each system. Two special cases were studied, the first case concerned with similar units, in the second case the two units are dissimilar with failure and repair time having exponential distributions. The results given in [25,26] have been derived as special case from the results of this chapter. Computer programs to calculate the MTSF and the profit of the two systems are done for the second special case for each system. From the comparison it has been concluded that the first system is more available than the second (see Appendices I.3 and I.2). The analysis and studies of this chapter has been accepted for publication in "Journal of Microelectronics and Reliability", England, 1992.

The fourth chapter considers the cost analysis of a two-unit warm standby system. The two units are identical. The operative unit and warm standby unit have different failure and repair time distributions. Initially one unit is operative and the other is kept as a warm standby. When the operative unit fails, the system waits for the repairman for some maximum

time, or until the other unit fails, when the warm standby unit fails, the system wait for the repairman until the other unit fails. On the failure of the second unit or on the completion of the maximum time, the system call the repairman immediately at the higher cost. The random variables concerning failure, repair, maximum waiting and arrival times are different and generally distributed functions .The MTSF, the availability analysis, cost analysis and expected profit in steady state are obtained .Three special cases were studied ,in the first case the failure time has exponential distributions and the repair time has a general distribution , in the second case the failure ,repair , maximum waiting and arrival times are exponentially distributed , while the third case concerned with the system having two units with cold standby . The results in [45] have been derived as a special case from the results of this chapter. Computer programs to calculate the MTSF and the profit of the system are developed for the second special case (see Appendices I.4 and II.3) . The study performed in this chapter has been accepted for publication in "Journal of Microelectronics and Reliability", England, 1991.

The fifth chapter considers two dissimilar units parallel system with facilities of preventive maintenance and inspection backed up by two types of repair. When both units are in normal operation, the system goes for preventive

maintenance at random epochs. Both units of the system can fail simultaneously due to common cause failure or they can fail one by one. The failure, repair, inspection and maintenance times are different and generally distributed. The MTSF, steady state availability of the system, expected busy period of the repairman, expected number of visits by the repairman and expected profit earned by the system are calculated. Two special cases were studied, the first case concerned with the failure, repair, inspection and maintenance times that are exponentially distributed while the second case concerning with the system without preventive maintenance. The results given in [14] have been derived as special case from the results of this chapter. Computer programs to calculate the MTSF and the profit of the system with and without preventive maintenance were developed from the first and second special cases. From the comparison, it could be shown that the first system is more available than the second (see Appendices I.5 and II.4). The analysis and studies of this chapter has been accepted for publication in "A M S E", France, 1993.

The sixth chapter discusses three systems for two dissimilar-unit cold standby system with three types of repairmen. In the first system, the repairman is always with the system. In the second system, the repairman comes immediately at the failure of a unit, while in the third