

AIN SHAMS UNIVERSITY  
FACULTY OF ENGINEERING  
STRUCTURAL ENGINEERING DEPARTMENT



**FINITE ELEMENT ANALYSIS OF MAT FOUNDATIONS  
RESTING ON NONLINEAR ELASTIC MEDIUM**

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B.Sc. Civil Engineering  
Ain Shams University, 1987

A Thesis  
Submitted in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science  
in  
Structural Engineering

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Cairo, 1993

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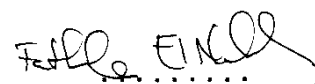
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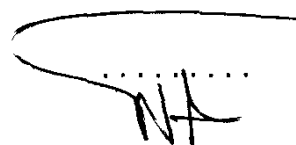
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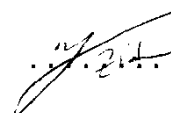
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## STATEMENT

This dissertation is submitted to AIN SHAMS University for the degree of Master of Science in Structural Engineering.

The work included in this thesis was carried out by the author in the Department of Structural Engineering, Ain Shams University, from October 1990 to October 1993.

No part of this thesis has been submitted for a degree or a qualification at any other university or institution.

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Ain Shams University  
Faculty of Engineering

Dept. of Structural Engineering  
Abstract of the M.Sc. thesis submitted by  
Eng. Mahmoud Abdel Hamed Hassanen

Title of thesis      Finite Element Analysis of Mat Foundations  
                         Resting on Nonlinear Elastic Medium.

Supervisors :    1. Prof. Dr. Hassan M. Hosny  
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Abstract :

This thesis studies the effect of nonlinear behavior of soil medium supporting mat foundations on the contact pressure distributions , internal force distributions and deformations in the mat foundations subjected to gravity loads.

soil medium has been represented by individual springs with constant modulus of elasticity at certain points of the mat or by infinite number (bed) of springs with variable modulus of elasticity. In each case, a soil mathematical model has been considered.

The theoretical assumptions utilized to deduce the stiffness matrix of rectangular plate in bending resting on bed of springs with variable modulus of elasticity has been presented and the stiffness matrix has been deduced. Also, a computer program is developed and checked to analyze mat foundations by the proposed method of analysis.

Different types of mat foundations such as mat with and without openings and mat with inverted beams has been analyzed using both the proposed method and the conventional method. A comparison study between both methods of analysis for contact pressure distributions, internal forces and deformations has been performed.

Keywords :

finite element method, plate in bending, nonlinear soil behavior, subgrade reaction model, nonlinear analysis, elastic model, mat foundations, raft foundations.

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