

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Ain Shams University
Faculty of Engineering



Analysis and Development of A Ballistic Missile Control System

By Engineer
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A thesis
submitted in partial fulfillment for the
Requirements of the degree of Master of Science
In Mechanical Engineering

621-5
S-4

5/4/9

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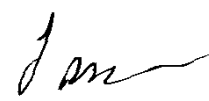
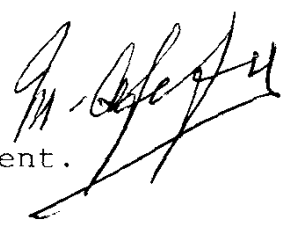
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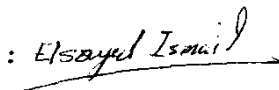
Statement

This dissertation is submitted to Ain Shams University for the degree of Master of Science in Mechanical Engineering.

The work included in this thesis was carried out by the author in the Department of Mechanical Engineering, Ain Shams University, from Dec. 1991 to Dec. 1994.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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ACKNOWLEDGMENT

I would like, first of all to express my indebtedness to both **Prof. Dr. Mohamed Yousef Afifi** and **Prof. Dr. Faraid Abdel-Aziz Tolba** Professors of Mechanical Engineering, Ain-Shams University, for opportunity to carry out this work. They are the supervisors of this work. I wish to express my sincere appreciation to them for continuous guidance, encouragement and constructive discussions.

I am especially grateful to my colleagues **Mr. T. Badawi, Mr. A. Abdel-Wahed** and **Mr. O. Gaweish** senior systems engineers at ABD company for helpful discussions and comments.

The help given by the colleagues in Systems Engineering Department is very much appreciated for the valuable discussion regarding this work.

El-sayed Ismail,
Cairo, 1995.

ABSTRACT

The problem of ballistic missile range accuracy represents the ultimate goal of missile autopilot designers. The errors taking place at any period of the flight time certainly accumulate and finally have a strong impact on the missile range at the end of the flight. Beside that, the lack of information due to the insufficient publications about the autopilot dynamics and channels interaction does not give the chance to the design engineers to diagnose the sources of the different errors and then introducing the effective solutions to these problems.

The effective performance of missile autopilot usually is not attained without an existing detailed analysis of the different components of the control system.

The main objective of the thesis is to introduce as much as possible a complete analysis of the missile control system including:

- 1- Missile dynamics represented in the form of nonlinear and linearized mathematical models and their validity.
- 2- Autopilot controller of the main channels (Pitch, yaw, roll and Lateral acceleration).
- 3- Autopilot final control element and its analysis.
- 4- Complete representation of the three main channels and their coupling.
- 5- Analysis and evaluation of the autopilot performance indices in the main channels and the effect of the external disturbances.
- 6- Introducing the effect of coupling between the different channels.

Introducing the previously given points in the thesis, makes it a good candidate for an information bank, which can be consulted by the engineering staff of missile autopilots.

Thesis Summary

The thesis is divided into six chapters and four appendices as follows:

Chapter one represents literature survey.

Chapter two deals with missile dynamics. It includes complete nonlinear and linearized equations of motion of flying object. Thus, the transfer functions of the basic modes of missile motion are attained. Hence, the analysis of every mode is done with respect to control actions and disturbance.

Chapter three is devoted to a detailed analysis of servomechanism dynamics. The analysis is conducted based on a complete linearized model taking into consideration the effect of nonlinearities.

Chapter four deals with analysis of missile autopilot on base of decoupled pitch, yaw, roll and lateral acceleration channels. A representation of autopilot configuration is presented. Hence, the evaluation of the performance indices of the main channels are attained. Also, the effect of external disturbances is introduced.

Chapter five deals with the analysis of control system channels coupling. It presents firstly the coupled mathematical model. The analysis of yaw and roll channels is done again based upon the coupled model of missile dynamics. The comparison between the results of both coupled and decoupled models is given.

Chapter six presents the analysis of the results and conclusions.

The presented work is based on the analysis of autopilot dynamics at every second during the missile trajectory of controlled regime. Furthermore, six critical and important cases are selected for detailed analysis of frequency and transient characteristics.

The cases represent the times at which:

- maximum value of angle of attack is occurred
- maximum values of aerodynamic coefficients are attained.
- maximum value of dynamic pressure times angle of attack is occurred
- maximum value of dynamic pressure is occurred.
- maximum static instability is attained.
- at the moment of engine cut-off.

At the end of thesis four appendices are added:

Appendix-A introduces the systems of axes and their transformations.

Appendix-B present the transfer functions of longitudinal dynamics of missile body.

Appendix-C introduces nonlinear perturbed model of longitudinal motion.

Appendix-D presents nonlinear model of servomechanism.

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