MAGNETIC PROPERTIES OF TYPE-II SUPERCONDUCTORS

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ABSTRACT

The present work deals with the study of magnetic properties of type-II superconductors. The thesis consists of five chapters. In the first chapter, a brief introduction of the history and the difficulties encountered by this study through its evolution, is given. In the second chapter a general theory of pinning of flux lines by macroscopic inclusions in a type-II superconductor is considered. By using the generalized spherical function (GSF) method, a modified local model of an isolated flux line (not necessarily straight) has been used to calculate the energy of a system of flux lines and a macroscopic ellipsoidal and/or spherical inclusions inside the superconductor. It is shown that one possible cause of the pinning is that an attractive image force exists at the inclusions boundary. This idea agrees with the general theory of flux pinning in type-II superconductors.

The properties of the distorted flux lines near the surfaces having different shapes, of type-II superconductors are considered in chapter-III. The different shapes considered are: quarter-space with two plane surfaces, a slab bounded with three plane surfaces, ellipsoidal and spherical solids. The distribution of the magnetic field and the current density inside and outside the superconductor are determined. The force on the different elements of a flux line is completely determined in terms of the total current density at the position under consideration. The Gibbs free energy of the system is also calculated and was put in a simple form giving rise to some conclusions of physical interest.

In chapter-IV two problems related to surface hysteresis in type-II superconductors were considered. The first problem concerns the surface effects due to a type-II superconducting wedge. The balance between the attractive image forces and the repulsive effect of the fluxoid, gives rise to a potential barrier near the surfaces. It is shown that far from the edge of the wedge the barrier disappears when the external field is greater than the superheating critical field. The maximum force per unit length of the flux line at the surface is calculated. The second problem deals with the solutions of force-free configuration fields for superconducting ellipsoid. The driving force in this configuration will be zero and transport currents can be carried without pinning.

Some discussion and conclusions, which are, of physical interest are given in chapter-V.

The thesis is concluded by five appendices, which we think useful, dealing in more details with the various mathematical problems involved in the chapters.

The contents of chapters-II, III & IV are being prepared for publication in three papers.

CHAPTER I

INTRODUCTION

The phenomenon of superconductivity was discovered by the Dutch physicist Kamerling Onnes in 1911(1). The classification into type-I and type-II superconductors was first suggested fourty years later by Abrikosov in 1952(2). This classification is conveniently characterized by the Ginzburg-Landau (GL) parameter $\kappa = \lambda / \xi$, where λ is the penetration depth of the magnetic field and ξ is the coherence length.

Type-I (or ordinary) superconductors are found to be pure metals such as Sn and In having $\kappa < 1 / \sqrt{2}$. In the superconducting state, these metals exhibit perfect diamagnetism. This means that, if a magnetic field is applied to a type-I superconductor, currents circulate on its surface and the magnetic field is completely screened from its interior (Meissner effect(3)). This exclusion of the external field, raises the free energy of the system and subsequently a sufficiently large magnetic field causes the metal to lose its superconductivity reverting it to its normal state. The critical field beyond which superconductivity disappears is denoted by H_c. The magnetization curve of type-I superconductors is shown in Fig.1.

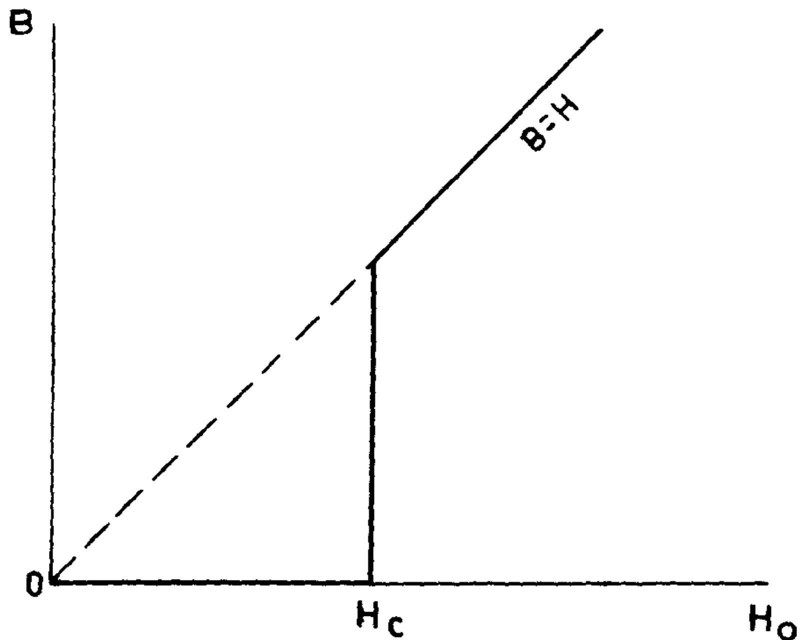

Fig.(1) The induction (or flux / cm^2) penetration behavior in type-I superconductors.

If $\varkappa > 1 / \sqrt{2}$ we have type-II superconductors which include many superconducting alloys and niobium metal. In type-II superconductors Meissner effect is also observed when the value of the applied magnetic field is lower than the first critical field H_{c_1} (see Fig. 2). Increasing the magnetic field superconductivity is conserved but Miessner effect does not hold. The magnetic induction increases and when the external magnetic field reaches the value of the second critical field H_{c_2}, the induction becomes equal to the external field. In the same time the superconductor is reverted to the normal state.

When the applied field strength H_o satisifies the inequality $H_{c_1} \ll H_o \ll H_{c_2}$ the type-II superconductor is said to be in the mixed state as was suggested by Abrikosov(4).

The main difference between type-I and type-II superconductors is that type-I materials exhibit the so-called intermediate state(5,6) whereas type-II materials exhibit the mixed state(5-7). In the intermediate state the normal regions are relatively thick, parallel laminar whereas in the mixed state they are vortex lines. As it was proposed by Abrikosov(4) the structure of the mixed state consists of a 2-dimensional periodic lattice(usually hexagonal) of flux lines.

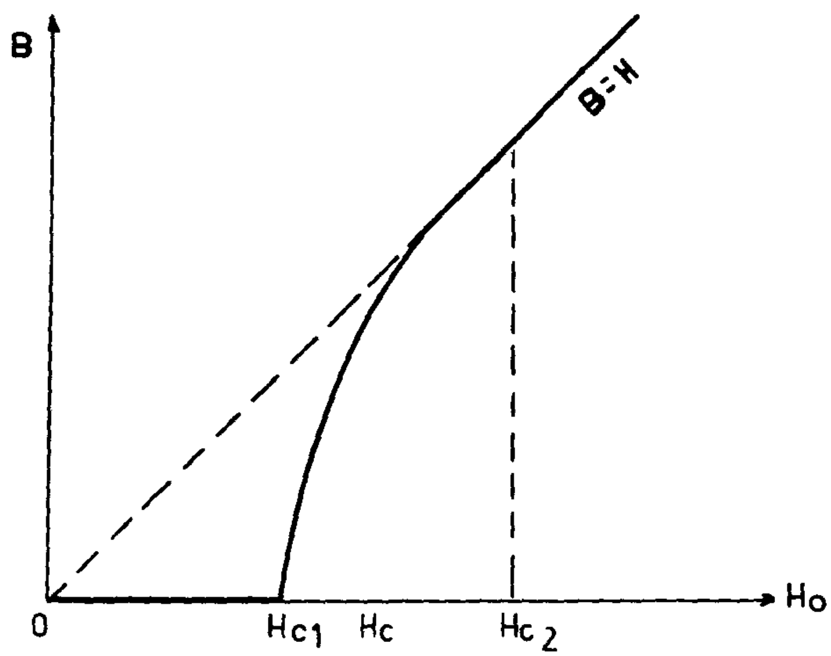

Fig.(2) The induction (or flux / cm^2) penetration behavior in type - II superconductors.

The magnetic behaviour of type-II superconductors was first studied by Abrikosov in 1957[4]. It was shown that if a type-II superconductor is placed in a sufficiently strong magnetic field, flux penetrates the superconductor in the form of flux lines(or current vortices, fluxoins,... etc). A flux line may be regarded, roughly, as a thin non-superconducting cylindrical core of radius ξ ; around this normal core and out to a distance $\sim \lambda$ circulate undamped superconducting currents. The magnetic field is maximum in the core but falls away to zero outside (Fig.3) . Each flux line lies parallel to the applied field and carries one flux quanta $\emptyset_o = (\pi \hbar c / e) = 2.07 \times 10^{-7}$ gauss cm^2, where $\hbar$ is Planck's constant divided by 2π , e is the electron charge and c is the velocity of light.

The vortices may be regarded as independent objects and one has to take into account their interaction with one another[8,9] and/or with the surfaces of the superconductor[10-20] and/or with the inhomogeneities[16,21].

In describing the mixed state quantitatively it is convenient to divide the field range between H_{c_1} and H_{c_2} into two regions: the first region $H_{c_1} < H_o \ll H_{c_2}$ is described by the so-called London model[22] (or approximation) whereas, the second region $H_o \sim H_{c_2}$ is described by the GL-theory[23].