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GRILLAGE ANALOGY FOR CRACKED R.C. SLABS

BY

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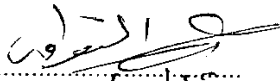
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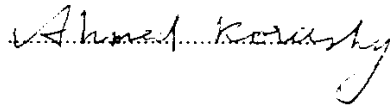
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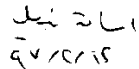
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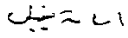
STATEMENT

This dissertation is submitted to Ain Shams University for the degree of **Master of Science** in Civil Engineering (structural) .

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DEDICATION

To my sweet wife and my sons Islam & Tarek

To my family: my beloved mother and father without whose love I could not have continued with this study.

To my friends, without their help, advice, and encouragement, this work would not have been possible.

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ABSTRACT

Name Usama Mohammed Nabil Nagy .

**Title “ Grillage Analogy for Cracked Reinforced
Concrete Slabs ” .**

**Master of Science Dissertation , Faculty of Engineering , Ain Shams
University .**

The thesis presents the response of Reinforced Concrete slabs after cracking and prior to failure using the grillage analogy . A simplified numerical method is developed to study their behavior up to failure using the stiffness method . The effect of the material non linearity due to change in the structural rigidity (flexural & torsional rigidities) is incorporated .

The flexural rigidity is represented by a trilinear moment-curvature relationship . The torsional rigidity is represented by a trilinear relationship taking into account the shear-torsion-flexure interaction . A computer program is prepared and various shapes of beams and slab are studied using the applied computer algorithm . The results have been compared with the published experimental data ; and , the load-displacement curves are plotted .The comparison has shown a good agreement . Thus , a simple approach has been presented to determine the ultimate load of slabs , this load is close to that obtained by the yield line theory .

**Key Words : reinforced concrete , beams , slabs , grillage analogy
, grids , ultimate , analysis , stiffness method , torsion .**

TABLE OF CONTENTS

Title	Page
Acknowledgment	i
Abstract	ii
Table Of Contents	iii
List of Figures	vii
List of Tables	x

CHAPTER 1 (INTRODUCTION)

1.1 General.	2
1.2 Review of Previous Work.	2
1.2.1 Significance of Nonlinear Analysis.	2
1.2.2 Grillage Analogy.	5
1.3 Objectives.	11
1.4 Scope.	11

CHAPTER 2 (METHOD OF ANALYSIS)

2-1 Introduction.	14
2.2 Stiffness Analysis of Grids.	14
2.2.1 Degree of Freedom.	15
2.2.2 Element Stiffness Coefficients.	15
2.2.3 Global Stiffness Coefficient For a Grid Member	20
2.3 Stress-Strain Relationships.	21
2.3.1 Steel Reinforcement.	21

2.3.2	Concrete in Compression.	23
2.3.3	Concrete in Tension.	25
2.4	Flexural Rigidity of R.C. Section.	26
2.4.1	Introduction.	26
2.4.2	Development of Moment-Curvature Relationship	28
2.4.2.1	Used Method to Calculate Moment - Curvature Relationship	29
2.4.3	Determination of Simplified Moment - Curvature Relationships.	34
2.5	Torsion of Structural Concrete.	36
2.5.1	General.	36
2.5.2	Torsional Stiffness in Homogeneous Sections.	37
2.5.3	Strength of R.C. Sections in Torsion.	38
2.5.3.1	Used Formulas To Obtain Properties of R.C. Sections Under Torsion	40
2.5.4	Torque-Twist Relationship.	43
2.5.5	Interaction Surface.	46
2.5.5.1	General About Interaction Surface	48
2.5.5.2	Flexure - Torsion Interaction	49
2.5.5.3	Shear - Torsion Interaction	55

CHAPTER 3 (CPOMPUTER PROGRAM)

3.1	Introduction.	60
3.2	Sequence of Analysis.	60
3.2.1	Linear Analysis Stage.	61
3.2.2	Calculation of Correction Factors Stage	63

3.2.2.1 Flexural Rigidity Correction Factors.	63
3.2.2.2 Torsional Rigidity Correction Factors.	66
3.2.3 Nonlinear Analysis Stage.	68
3.2.3.1 Nonlinear Technique.	68
3.2.3.2 Iterative Procedure.	69
3.2.4 Final Results Stage.	70
3.3 Computer Program.	72
3.3.1 Summary of Sub. Programs.	72
3.3.2 Limitations of The Program.	74
3.3.3 Flow-Chart of Main Program.	75

CHAPTER 4 (APPLICATIONS)

4.1 Introduction.	79
4.2 Test of Computer Program.	79
4.2.1 Check of Linear (First) Stage.	85
4.2.2 Nonlinear Stage Results.	85
4.2.2.a Check of Concrete Sections Properties.	85
4.2.2.b Check of Final Stage Results.	87
4.3 Applications.	92
4.3.1 R.C. Beams Under Pure Torsion	92
4.3.2 R.C. Beams Under Bending Moment only.	95
4.3.2.1 Moment-Curvature Relationship.	95
4.3.2.2 Simple Beam Under Concentrated Loading.	99
4.3.3 Reinforced Concrete beams in Torsion and Bending....	99
4.3.4 Ultimate Load of R.C. Slabs.	103
4.3.4.1 Case of One way Slab	103

4.3.4.2	Case of Flat Slab with Various Supporting Conditions.	107
4.3.4.2.1	Application (1)	107
4.3.4.2.2	Application (2)	114
4.3.4.2.3	Application (3)	120
CHAPTER 5 (CONCLUSIONS & RECOMMENDATIONS)		
5.1	Conclusions.	127
5.2	Recommendations for future researches.	128
APPENDIX REFERENCES		129
APPENDIX (A) : Input Data For Program Grid		134
APPENDIX (B) : Input Data For Program CV123T		144
APPENDIX (A) : List of Program Grid		145
APPENDIX (B) : List of Program CV123T		165

LIST OF FIGURES

Figure	Title	Page
Fig. 1.1	Concrete Compressive Stress Block Parameters	4
Fig. 1.2	Proposed Stress-Strain Relationship for confined or Unconfined Concrete	4
Fig. 1.3	Stress-Strain Curves for Steel Reinforcement	4
Fig. 1.4	Grid Model	6
Fig. 1.5	Description of the Slab as a Grid System	6
Fig. 1.6	Idealized Elastic - perfectly Plastic Moment Curvature Relationship	8
Fig. 1.7	Piecewise Linear Load - Deflection Curve	8
Fig. 1.8(a)	Simplified Moment - Curvature Relationship	10
Fig. 1.8(b)	Simplified Torque - Twist Relationship	10
Fig. 2.1	Degrees Of Freedom At Joints of Grid Element	16
Fig. 2.2(A)	Member Stiffness :Unit Translation and Rotation At J .	17
Fig. 2.2(B)	Member Stiffness :Unit Translation and Rotation At K	18
Fig. 2.3	Stress-Strain Curves For Steel Reinforcement	22
Fig. 2.4	Proposed Stress-Strain Relationship For Confined or Unconfined Concrete	22
Fig. 2.5	Idealized Stress-Strain Relationship For Concrete In Tension	22
Fig. 2.6	Flexural Deformation of Conc. Element	27
Fig. 2.7	Proposed Stress-Strain Relationship For R.C. Sections .	31
Fig. 2.8	Simplified Moment - Curvature Relationship	35
Fig. 2.9	Torsional Stress In Homogeneous Sections	39
Fig. 2.10	Torsional Rigidity For T-Sections	39
Fig. 2.11	Torque - Twist Curves	44
Fig. 2.11(D)	Torque - Twist Curves Of Beams With Various % Of Rft.	45

Fig. 2.12	Interaction surfaces for combined bending-shear-torsion	47
Fig. 2.13	Mode Failure for a concrete section with stirrups.	50
Fig. 2.14	Mode Failure of T-beams	51
Fig. 2.15	Interaction curves for combined bending-torsion	56
Fig. 2.16	Shear- torsion interaction curve	58
Fig. 3.1	Local System of Loads at Ends of Grid element	62
Fig. 3.2	Iterative or / Newton Procedures	71
Fig. 3.3	Flow chart of the main program	76
Fig. 4.1	Schematic Plan of Six - Panel Waffle Slab	81
Fig. 4.2	Schematic Cross Sections of The Model and Details of Reinforcement	82
Fig. 4.3.a	Cases of Loading	83
Fig. 4.3.b	Scheme of Loading to Failure (Load Patterns I & III). ..	84
Fig. 4.4	Comparison Load - Deflection Curves Between the Exp. Data and Developed Method Results (case 1 of Loading). ..	89
Fig. 4.5	Comparison Load - Deflection Curves Between the Exp. Data and Developed Method Results (case 1 of Loading). ..	90
Fig. 4.6	Comparison Load - Deflection Curves Between the Exp. Data and Developed Method Results(case 1 of Loading). ..	91
Fig. 4.7	Details of Tested R.C. Beam Under Pure Torsion	94
Fig. 4.8	Comparison Between the Experimental Data Tested by [17] and Developed Method Results.	96
Fig. 4.9.a	Rect. R. C. Beam Tested By Mcneice [14]	100
Fig. 4.9.b	The Actual Rectangular Section of The Beam And Suggested T-Section	100
Fig. 4.10	Comparison Load - Deflection Curves Between the Exp. Data by Tested [14] and Developed Method Results.	101
Fig. 4.11	Comparison Load - Deflection Curves Between Actual Rect. Section And Suggested T-Section	102