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# THE RESTRICTED THREE-BODY PROBLEM FOR MOTION IN THE PLANE OF CIRCULAR ORBITS

## A THESIS

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By

**MOHSEN HANAFI MOHAMED MOSSA**

Mathematics Department  
Faculty of Education  
Ain Shams University

18446



Supervisors

**Dr. A. EL-MAGD A. MOHAMED**

Professor of Mathematics  
Faculty of Education  
Ain Shams University

**Dr. FAWZY M. F. EL-SABAA**

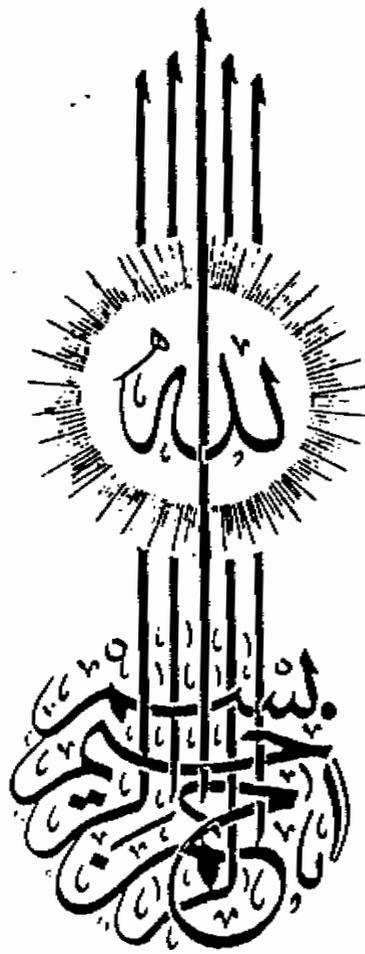
Lecturer of Mathematics  
Faculty of Education  
Ain Shams University

**1984**



# **TO PARENTS**





وما توفيقي إلا بالله

صدق الله العظيم

M. SC. COURSES  
STUDIED BY THE AUTHOR (1980-1981)  
AT AIN SHAMS UNIVERSITY  
FACULTY OF SCIENCE

- 1 - *Elasticity.*
- 2 - *Numerical Analysis.*
- 3 - *Classical Mechanics.*
- 4 - *Magnetohydrodynamics.*
- 5 - *Fluid Dynamics.*
- 6 - *Theory of Stability.*

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# SUMMARY

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## SUMMARY

Our aim in this thesis, which consists of four chapters is to study the restricted three bodies problem

In chapter (I) , we presented an introduction to the following topics:

- 1 - Comparison between Celestial Mechanics and Space Mechanics.
- 2 - The approaches to the understanding of the behavior of Dynamical systems.
- 3 - The history of the restricted problem, which was organized by a round of three names Euler, Jacobi and Poincaré.

In chapter (II) an exhibition of the restricted three body - problem with a complete survey

The planar restricted three bodies problem with circular orbit are exhibited. And the equations of motion can be expressed by Hamilton - Jacobi equations. We proved that the given system belong to Liouville's system under a certain condition that is the two massive bodies which draw a circular orbits about their centre

of mass are of equal masses. So, the variables are separated and the system reduced to quadratures.

In chapter (III) the lines of surfaces of the two functions  $Q(\lambda)$ ,  $G(\mu)$  [ where  $\lambda, \mu$  are the elliptic coordinates] were established. The functions  $Q(\lambda), G(\mu)$  are obtained by separating the Hamilton - Jacobi equations in the case  $m_0 = m_1$ , in the second chapter.

The singular points of the two functions obtained. The types of these points were hyperbolic or elliptic points which are studied in details .

And in chapter (IV) we discussed the restricted problem of three bodies when one of the two massive bodies is rigid... the domains of possible motions obtained by finding the curves of zero velocity, the periodic orbits are found by solving the differential equations of the given system numerically by using the method of Poincaré... we classified these orbits and we drew these classes in the plane  $(C, x)$  where  $C$  is the constant of energy. We studied the stability of these periodic orbits also numerically and the regions of stability to these orbits were founded.

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**CHAPTER I**

**INTRODUCTION**

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Celestial mechanics [1] is the branch of astronomy that is devoted to the motion of natural celestial bodies while space mechanics is the branch of space sciences that is devoted to the motion of artificial celestial bodies. Both activities can be looked at as branches of dynamics or applied mathematics; Celestial mechanics is oriented mainly towards scientific aims while space mechanics, in addition to scientific interest, is also concerned with the solution of engineering problems. The scientific objectives of space explorations cannot be accomplished without space mechanics activities, which latter in turn rely heavily on celestial mechanics. Certain problems and techniques are common to celestial and space mechanics and these receive naturally great emphasis at the present time. The outstanding example is the problem of three bodies which is at common focal point of celestial and space mechanics.

This thesis is devoted to treating some points in the restricted three bodies with circular orbit.

[I.1] *The approaches which help in understanding the behavior of dynamical systems.*

The purpose of dynamics is to characterize the totality of possible motions of a given dynamical system. Such a characterization does not necessarily mean an explicit, closed - form, general solution of the problem since this is seldom possible, and when it is possible, it is most of the time neither meaningful nor helpful in understanding the behavior of the system. An example is the problem of two bodies, which is considered solved since the properties of the totality of possible motions are known. Although the coordinates describing the motion of the bodies participating in the problem cannot be represented as explicit functions of the time in closed form, the problem is nevertheless considered solved.

Qualitative, quantitative, and formalistic dynamics are the three major approaches to the understanding of the behavior of dynamical systems. The qualitative approach is probably the most elegant and sometimes the most powerful. The formalistic method is the basis of classical celestial mechanics. The quantitative approach is often the most popular among