DISCRETE SEQUENTIAL ESTIMATION OF BIT SYNCHRONIZATION IN DETECTION

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Nomenclature :

θ	=	bit timing misalignment
T	=	the locally generated bit period
T_r	=	the incoming bit period
T_o	=	the sampling period
f	=	the local clock frequency
f_r	=	the incoming clock frequency
Δf	=	the frequency difference between oscillator
$\zeta = \frac{\theta}{T}$	=	normalized bit timing misalignment
$z(t)$	=	the observation
$x(t)$	=	the received binary modulated process
$s_1(t), s_2(t)$	=	the transmitted waveform
$n(t)$	=	additive Gaussian distributed noise
E_s	=	average signal energy in a bit period
E_d	=	average energy in the signal derivative
N_o	=	noise spectral density (single side band)
$\frac{E}{N_o} = \frac{S}{N_o}$	=	signal to noise ratio
K	=	Total number of samples in one bit period
L	=	Total number of bits considered
$\tilde{\theta}$	=	the error in bit timing
$d(k)$	=	time drift
Q	=	variance of the drift
$R \equiv R_n$	=	the variance coefficient of noise
$R_{\dot{n}}$	=	the variance coefficient of the derivative of noise
$Z(k)$	=	the observation vector

GENERAL INTRODUCTION

In an interesting class of communication systems, information is transmitted by digital modulation of waveshapes of the form

$$s(t) = f(t) \sin (w_o t + \theta) \qquad (1)$$

(f(t)) is a pseudorandom time function, i.e., it is generated deterministically by means of a code of a long period, but it appears to be a random wave to an observer who has available only a segment of (f(t)) and who is ignorant of the code. The spectrum of (f(t)) consists of frequencies much lower than (w_o). In this class of communication systems (f(t)) is called the subcarrier.

Actually (f(t)) is usually a binary encoding of the information (for instance, by multiplication of the ingredient signal form by plus or minus one, for a length of time called the baud length (the bit period) corresponding to mark and space, respectively). In order to extract the information from the modulated transmitted wave, the receiver must be able to generate periodically the ingredient signal, correlated it with the transmitted modulated waveshape for the duration of a single mark or space signal and consider the sign of the locally generated signal as the polarity of the modulation.

There are, of course, other ways to modulate the carrier (frequency shift, for instance, in which (f(t)) is made to modulate the carrier frequency). In any case, signal reception is achieved through binary detection (the testing of an appropriate simple hypothesis vs. the appropriate single alternative).

If a maximum likelihood receiver is used and the background noise is white, the initial step in the detection process is the formation of the correlation of the transmitted waveshapes with the expected unmodulated waveshape, the latter being internally generated at the receiver. In order to perform the desired correlation, the receiver must be able to generate a perfect replica of the unmodulated transmitted wave. The receiver has available some information about the transmitted waveshaped (1). (e.g. the basic signal form, the bit period, etc...). If the reception is to be coherent, the receiver must know:

1) The carrier frequency and phase.
2) The bit rate and duration.

If the receiver is moving with respect to the transmitter the carrier frequency is subject to Doppler shift, while the bit phase depends on the distance between the receiver and the transmitter.

From the above it is clear that the synchronization of the receiver to the received signal is of utmost importance. In some systems such synchronization is achieved from the transmitter, while in other systems the receiver extracts from the signal sufficient information necessary for synchronization. In such systems, synchronization is basically the estimation of two parameters; carrier frequency and the bit rate. But for coherent reception, the receiver must estimate also the carrier or bit phase.

A panel discussion at the IEEE Winter General Meeting (1) in 1963, focussed attention on several fundamental questions regarding synchronization, and noted general deficiencies in the amount of effort devoted towards this aim. In connection with bit timing, it is convenient to categorize the approaches into two broad classes:

1) Methods in which the format for all or part of the transmitted information conforms, for synchronization purposes, to a prescribed pattern (transmitted reference systems (TR)).

2) Techniques in which the timing information is extracted from message data (self synchronized system (SS)).

In this latter class, two commonly employed means of synchronization are:

1) Maximum likelihood detection with parameter estimation (see for example (2)).

2) Phase-lock loop (PLL) tracking of the bit frequency spectral component in the demodulated message (3), (4).

Very recently sequential estimation methods have been introduced by Sage and McBride (5), (6). The present study is an extension of their work in the sequential estimation of bit synchronization.

In this thesis, the bit synchronization problem will be discussed thoroughly from the point of view of the control theory. Chapter (I) begins with the formulation of the problem, the optimum and some suboptimum receivers are discussed. In Chapter (II) the work of McBride and Sage in this field is reviewed and their results are reinvestigated. Chapter (III) contains a trial to solve the difficulties which are arised in their work by making use of the extension of the discrete maximum principle. Finally Chapter (IV) is concerned with the bit synchronization problem as a two dimensional one in order to overcome the disadvantages of the previously mentioned schemes, and recursive bit estimation algorithms are found. Further more, the Cramer-Rao bound of the variance of error is applied for the bit sync. problem in both one and two dimensional cases. During all this work the same example, which is given by McBride and Sage (5),(6), is taken in order to establish a solid base of comparison.

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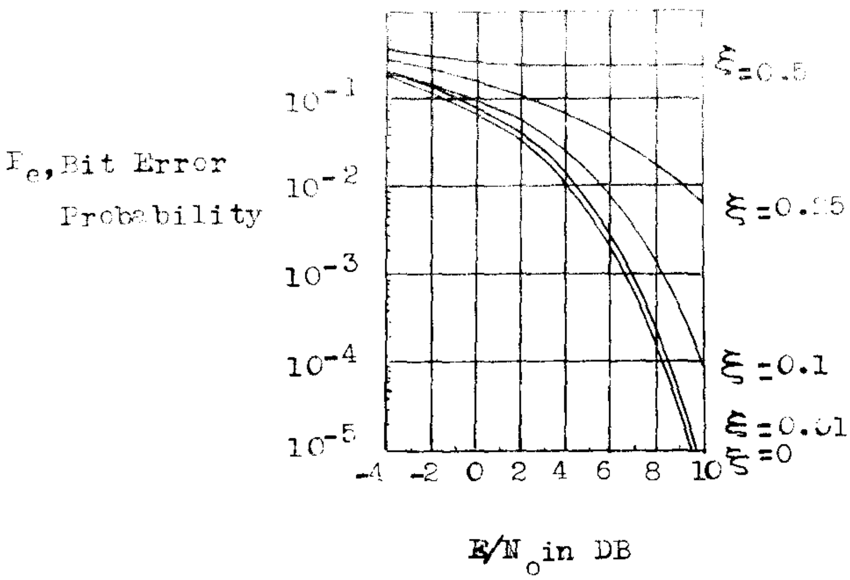
I. OPTIMUM ESTIMATION OF BIT SYNCHRONIZATION

I.1. Introduction

In digital communication systems, the theory of optimum data detection requires that precise knowledge of the bit transition time be known to the receiver before a bit-by-bit detection can be implemented. The optimum receiver for a phase shift keying (PSK) modulated waveform embedded in additive Gaussian noise is the matched filter or the correlation detector. This device is usually implemented by an integrate and dump circuit which requires the bit timing of the incoming sequence. This timing is supplied by the bit synchronizer. SS systems, which acquire bit synchronization timing directly from the incoming modulated data sequence, are generally required for most applications. The majority of the present-day bit synchronizers are designed to operate on the noisy baseband analog data. This may not be the best technique for a completely digitalized receiver since it requires that the sampled (IF) or carrier data be used to estimate the carrier and/or subcarrier frequency and phase of the incoming sequence. The resulting estimate is then used to convert the received waveform to an analog baseband process. Although the idea of converting from analog to digital signals, estimating frequency and phase, then converting back to an analog baseband process may not be too appealing from the digitized receiver viewpoint, it does place the bit synchronization estimation problem within the framework where a large amount of research has been performed. Furthermore, this conversion to an analog baseband process can probably be avoided while still using the available theory and digital implementation techniques developed for the baseband model.

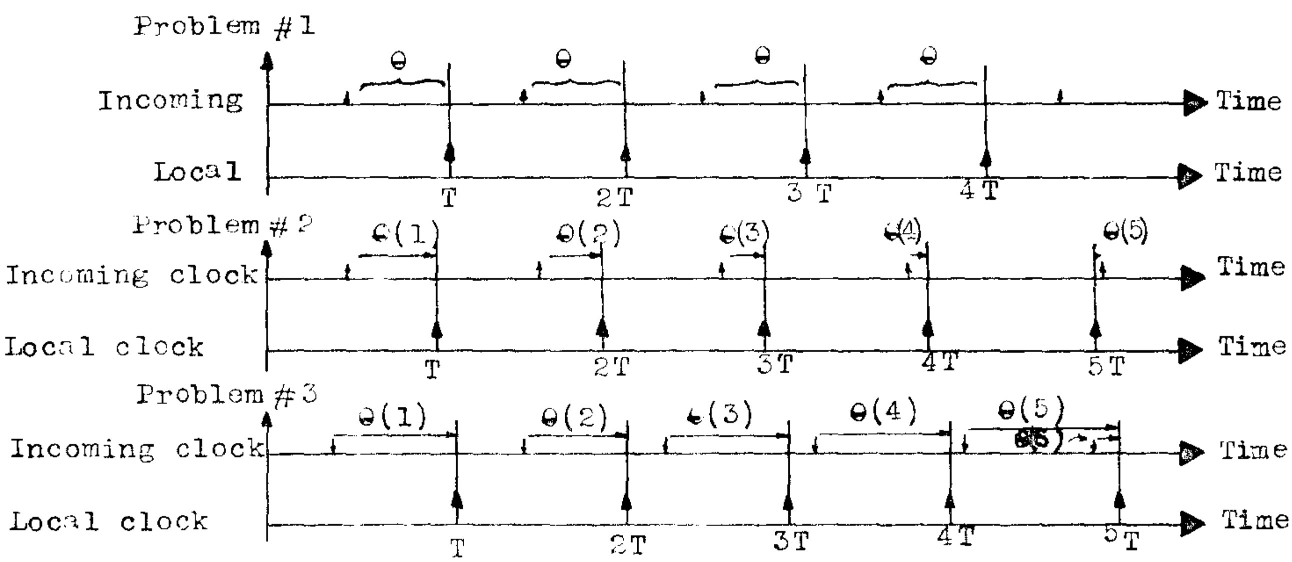
Two parameters are generally used to measure the performance of bit synchronizers, the r.m.s. jitter or timing misalignment and bit slippage. Which of these is more important depends on the type of data being transmitted and the type of modulation employed. Probably the most widely used performance parameter for digital communication systems is the bit error probability (BEP) versus the average bit energy per noise density, E/N_o. Degradation in BEP performance can be directly related to the bit timing misalignment. On the other hand, the effect of bit slippage is not easily related to the BEP performances and is usually related to how it affects the word synchronization.

A conditional BEP may be derived for PSK modulated data in the presence of additive white noise which is conditioned upon a static time misalignment θ. This BEP (defined by p_e) is easily shown (7) to be given by



P_e, Bit Error Probability
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\xi = 0.5$
$\xi = 0.25$
$\xi = 0.1$
$\xi = 0.01$
$\xi = 0$
-4 -2 0 2 4 6 8 10
E/N_0 in DB

Fig. I.1



Problem #1
Incoming
Local
Time
Time
T 2T 3T 4T
Problem #2
Incoming clock
Local clock
Time
Time
T 2T 3T 4T 5T
Problem #3
Incoming clock
Local clock
Time
Time
T 2T 3T 4T 5T

Fig. I.2

is then centered around the stability of the oscillators (clocks) which generate the bit periods, the initial phase and frequency. (Phase as used here is defined from the timing misalignment parameter θ as $2\pi\theta/T$ where $\theta = \rho T$). The characteristics associated with these can be divided into three problems.

Problem 1:

frequencies completely stable and their values known and initial phase unknown.

Problem 2:

frequencies have deterministic slow drift characteristics; the initial frequencies unknown to within bounds and initial phase unknown.

Problem 3:

frequencies have random drift characteristics (slow drift) about mean frequencies; initial frequencies unknown and initial phase unknown.

Division of the clock characteristics into these three problem areas covers the range of possibilities and also, divides the degree of complexity of the bit synchronization problem from the simplest, problem 1, to the most complex, problem 3. These three problems are illustrated in Fig. (I.2) where the unknows associated with both clocks are characterized by of (Θ) conditioned on the observation. Hence if (z) is the observation, then the conditional cost becomes:

$$C = \int_{\Theta} c(\Theta)\ p(\Theta/z)\ d\Theta \qquad ..(I.3.1)$$

The functional ingredient required in Bayes parameter estimation is the a posteriori density $p(\Theta/z)$ and the first step in any Bayesian parameter analysis (such as the biy sync. estimation problem) is to determine $p(\Theta/z)$. For the bit sync. estimation problem, the a posteriori density $p(\Theta/z)$ and the observation models are:

$$p(\Theta/z) = \frac{p(z/\Theta)\ p(\Theta)}{\int_{\Theta} p(z/\Theta)\ p(\Theta)\ d\Theta} \qquad ..(I.3.2)$$

$$z(t) = x(t,\Theta) + n(t) \qquad ..(I.3.3)$$

where (Θ) and (z) are symbols used for simplicity and where $p(\Theta)$ is the a priori density for the time uncertainty (Θ). The maximum a posteriori (MAP) estimate algorithm would be to find $a(\hat{\Theta})$ contained within its range of definition such that $p(\Theta/z)$ is maximized. This is equivalent to:

$$\max_{\hat{\Theta}} p(\Theta/z) \longrightarrow \max_{\hat{\Theta}} p(z/\Theta)\ p(\Theta) \qquad ..(I.3.4)$$

because the denominator of Eq. (I.3.2.) is not a function of (Θ). The parameter (Θ) to be established is defined as the time uncertainty from the start of the observation to the first bit transition point. The bit transition point is that