HYDROMAGNETIC STABILITY OF CYLINDRICAL ANNULAR FLUID JET

THESIS

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By

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SUMMARY

SUMMARY

In this thesis we have studied some interesting stability problems which have crucial applications in science and some results are in good agreement with the Kendall's experimental work (1986). Chapter 1 is an introduction.

In chapter II, the instability of a streaming annular magnetised fluid jet surrounding a solid mantle subject to capillary, inertia, fluid kinetic pressure gradient and Lorentz forces has been developed for all kinds of perturbations. Several recent works are recovered as limiting cases from the present result. The analytical results are confirmed along with numerical methods. The streaming has a strong destabilizing influence in hydrodynamic and hydromagnetic versions. That influence is found to be independent of the kind of perturbation and idem the stabilizing effect of the uniform magnetic field pervaded interior the annular fluid jet model. The longitudinal field penetrated exterior the model is strongly stabilizing in both axisymmetric and non-axisymmetric perturbation modes. The exterior transverse varying magnetic field is destabilizing or stabilizing according to restrictions. Large value for the parameter of the transverse field is needed so that the stability due to this transverse field may arise and thence the normal stability states are modified. The streaming destabilizing influence may be overcome if the intensity of the axial fields interior and exterior the model are so strong enough such that the Alfvén magnetic speed is much larger than the magnitude of the basic streaming velocity of the fluid in the unperturbed initial state. These results are verified numerically via computer for different problem's parameters.

This work has been published in the *International Journal "Physica Scripta" Volume __56__, (1997) 193-199.*

In Chapter III, we study the magnetodynamic stability of streaming hollow jet pervaded by two dimensions varying magnetic field. By a hollow jet it is meant that a gas cylinder embedded into an infinite liquid. That model is the mirror case of the full liquid jet in a gas medium of negligible motion. This model has many practical applications in several domains of science, so it is deserved to be studied.

The problem is formulated and the MHD basic equations are solved in the initial and perturbed states. Upon applying appropriate MHD boundary conditions, the net solutions are identified and moreover the eigenvalue relation is established. The (in-) stability states are discussed analytically and numerically.

Chapter IV, is devoted to elaborating the instability of a gas cylinder in a moving finite liquid with double perturbed interfaces under the action of capillary and inertia forces. The motion of the liquid has always a destabilizing influence, which increase the region of instability. But the longitudinal wavenumber of the maximum instability does not change with the liquid velocity. The liquid-gas cylinders radii ratio plays an important role in the stabilizing of the annular jet. The present results are in good agreements with the experimental results of Kendall (1986).

The results of this problem have been published in the *Int. Conf. at the Faculty of Science, Al-Azhar University (Cairo) 1997, p. 106.*

In chapter V, the hydromagnetic stability of a hollow jet (radius R) is presented analytically; the results are confirmed numerically and interpreted physically. A general eigenvalue relation describing the (in-) stability characteristics of the model is derived.

In the absence of the magnetic field, the model is absolutely unstable only to axisymmetric perturbation whose wavelengths are