HYDROMAGNETIC PERTURBATIONS IN AN IDEAL FLUID OF FINITE ELECTRICAL CONDUCTIVITY

A Thesis

Submitted in Partial

Fulfillment of the Requirements

for the Award of The Master Degree

in Science (Mathematics)

BY

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1990

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OF FINITE ELECTRICAL CONDUCTIVITY

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HYDROMAGNETIC PERTURBATIONS IN AN

IDEAL FLUID OF FINITE ELECTRICAL

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Besides the work carried out in this thesis, the candidate has attended and past successfully the following post-graduate courses:

1. Magnetohydrodynamics
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ACKNOWLEDGEMENTS

I am deeply grateful to Professor Dr. F. A. AYOUB for accepting the supervision of this work and for his valuable encouragement.

I would like to express my deepest appreciation to Dr. M. K. ABDEL HADY for her valuable suggestions and discussions during the solution of the problem.

Finally my sincere thanks to Professor Dr. S. A. SHERIF head of the Mathematics Department, University College for Women, Ain-Shams University for beneficial help.

Ain-Shams University

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"Hydromagnetic perterbations in an ideal fluid of finite electric conductivity".

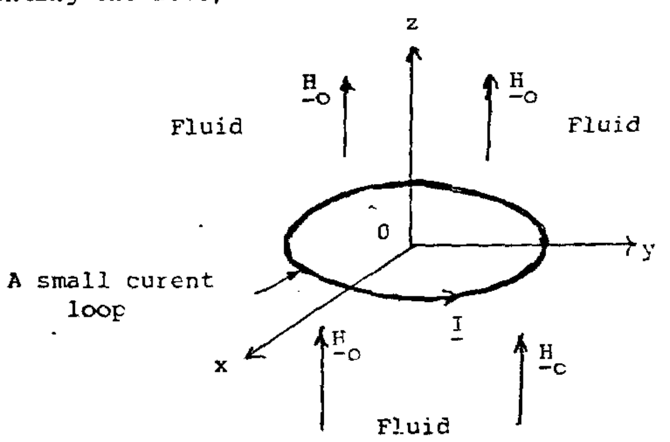
Thesis submitted in partial flulfilment for the requirements of the award of M. Sc. degree (Applied Mathematics)

by

Salwa Mohamed Gamal El-Din El-Mohandis

The aim of this work is to add more information about what is called the secular variation (S.V.) of the Earth's magnetic field. This will help to add more light on the origin of the geomagnetic field, a problem which has not yet been solved.

The model chosen is a magnetic dipole $\underline{M}$ suddenly introduced in an ideal unbounded fluid of finite elctrical conductivity where a homogeneous magnetic field $\underline{H}_o$ is prevailing parallel to the dipole axis. Asymptotic solutions of the resulting perturbations $\underline{h}$ (representing the S.V.)



of the homogeneous field $\underline{H}_o$ and of the fluid velocity $\underline{U}$ have been obtained in the two following extreme cases:

(i) Short times, (ii) Long times.

Maxwell's equations and the Eulerian hydrodynamic equations are used; the magnetic dipole is replaced by a small current loop of current density $\underline{I}$, where

$$\underline{I} = \underline{1}_\phi \; I \, \delta(\rho-a) H(t).$$

Laplace transformations in time, a Hankel transformation in space have been applied as well as the asymptotic expansion of the Hyper Geometric function ${}_1F_1$ for large values in the form

$${}_1F_1(a;b;-x) = \frac{\Gamma(b)}{\Gamma(b-a)} \, x^{-a} \; .$$

To obtain the asymptotic solutions of the problem, the two following important results have been proved and applied for large values of time, t,:

(1) The inverse Laplace transform

$$L^{-1} \frac{p^{r/2}}{(a+p)^{\nu}} = \frac{t^{-r/2}}{a^{\nu}\Gamma(1-\frac{r}{2})}$$

(2) The integral transform

$$\int_0^\infty \xi^\mu \, e^{-\xi^2 z^2/4t} J_1(\xi\rho)\, d\xi = \begin{cases} 1 & (\mu=-1) \\ \frac{1}{\rho} & (\mu=0) \\ \frac{1}{\rho^2} & (\mu=1) \end{cases} \; .$$

These asymptotic expansions helped to achieve a simple but powerful method for solving the problem.

The thesis contains an introduction and five chapters.

Chapter 1: Contains the basic equations of the problem.

Chapter 2: Contains introducing the source of disturbance; the boundary conditions at the loop; and some other results applied in solving the problem.

Chapter 3: Contains solutions of the problem for 4 different cases of short time.

Chapter 4: Contains solutions of the problem for 4 different cases of large values of time.

Chapter 5: Contains

(i) Explanation for the special method used in drawing the charts.

(ii) Physical Discussion of the problem.

Supervisors:

Prof. Dr. F. A. Ayoub

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CONTENTS

SUMMARY

The problem of hydromagnetic disturbances in an unbounded, ideal fluid, of finite electrical conductivity, due to the sudden introduction of a magnetic dipole, where an excitation field $\underline{H}_o$ exists, has been discussed.

The resulting perturbations, $\underline{h}$, of the magnetic field and of, $\underline{U}$, the fluid velocity have been discussed in the two following extreme cases:

(i) Short times.

(ii) Long times.

Asymptotic expansion for large values of time of the confluent hypergeometric function, F_1, has been used. Inversion of some Laplace transformations, as well as an expression for the integral representation

$$\int_o^{\infty} \xi^{\mu} e^{-\xi z/4t} J_1(\xi\rho)\, d\xi$$

have been derived for large values of time.

These asymptotic expansions allow us to achieve a simple but powerful method for solving the problem.

The thesis contains an introduction and five chapters:

Chapter 1: Contains the basic equations of the problem.

Chapter 2: Contains the source of disturbance, boundary conditions and some results applied in solving the problem.

Chapter 3: Contains solutions of the problem for short times.

Chapter 4: Contains solutions of the problem for long times.

Mathematical representations of the magnetic disturbances $\underline{h}$ as well as of the fluid motion velocity $\underline{U}$ are obtained. Four cases are discussed for each of short and long times.

Special method is used to draw the diagrams representing the obtained perturbations.

Chapter 5: Contains the method used in drawing the charts as well as physical Discussion of the problem.

INTRODUCTION

INTRODUCTION

HISTORICAL NOTE

The geomagnetic field:

The problem of how the Earth is magnetized has been considered as a baffling riddle of some hundreds of years. Eeven now the theory of the existence of terrestial magnetism, is by no means, has not got a satisfactory explanation. There is not till now a single theory to explain successfuly the axial dipole field and the associated secular (S.V.) field.

It was early known that a magnetic needle not only tended to point north, but if free to move vertically, it would also dip in the Northern Hemisphere and point above in the Southerm Herisphere. The first man to gain a true conception of the general character of the Earth's magnetic field, seems to have been William Gilbert of Calchester, Queen Elizabeth's physician. In 1600 be described in his treatise "De Magnete" a simple model experiment. He cut a sphere of lodestone and examined the distribution of direction of the magnetic force over its surface with a tiny magnetic needle freely pivoted. He showed that the angle of dip varied with the distance from the poles of his model in approximately the same way as the angle of dip observed on the Earth. He concluded that the Earth itself is a great magnet, similar to his magnetised shpere except in size; and also its magnetic influence proceeds from within , thus cleared away the belief that the force which effects the compass needle is found in the atmosphere or in the stars as has widely believed in that time.

The next important advance was when the german mathematical physicist K.F. Gauss (1838) assumed that there are no electric currents or permanently magnetised material near the Earth's surface and therefore that the magnetic field can then be expressed as the gradient of a single valued potential which satisfies Laplace's equation, and therefore it can be expressed as a general solution in a series of spher ical harmonics. Gauss' work has been repeated since. The complete anlysis of the observed values of the components of the geomagnetic field for the year 1885 have been carried out by Schmidt (1895); by Dyson and Furner (1923) for the year 1922; by Vestine et al (1947) for the year 1945 and an additional one for the same year by Alfonasieva (1946).

It is to be noted that, although sphereical harmonic analysis is a satisfactory procedure to give a good representation of the main field over the Earth's surface alone in terms of an axial dipole at the Earth's centre, yet it fails to determine the field at great depths, where the regular and irregular fields may be of almost of equal magnitude.

Therefore any theory of the geomagnetic field must give an explanation for the axial main dipole field as well as its secular variation field.

The starting point of modern development of the subject was marked, just before the second world war, when Elsasser (1939) was