

Handwritten signature and date: 13/7/78

**CRITICAL REVIEW ON EQUATIONS DESCRIBING  
YIELD RESPONSE TO WATER AND NUTRIENT  
APPLICATION**

Handwritten signature: 13/6/78

**By**

**AHMED ABD EL FATTAH AHMED IBRAHIM**  
B. Sc. Agric. ( Soil Science ) Ain Shams University 1975

**THESIS**

*Submitted in Partial Fulfilment of the Requirements for the Degree*

*Of*

**MASTER OF SCIENCE**

**In**

**SOIL SCIENCE**

Handwritten notes: 13/1/78, A A

Soil Department  
Faculty of Agriculture  
Ain Shams University  
Cairo



**1978**



APPROVAL SHEET

Name : Ahmed Abd El-Fattah Ahmed Ibrahim

Title: Critical review on equations describing yield response to water and nutrient application.

Thesis submitted for the degree of  
Master of science  
in  
soil science

This thesis has been approved by :

-----  
A. M. Galal  
-----  
M. A. Omar  
-----  
M. Y. El Awady ,  
-----

Date:        /        / 1978.        committee in charge



### ACKNOWLEDGMENT

The author wishes to express his appreciation and deep gratitude to Prof. Dr. **T.EL**-Kobbia, Soils Department, Faculty of Agriculture, Ain Shams University and Prof. Dr. Eng. M.N. El-Awady, Soils Department, Faculty of Agriculture, Ain Shams University for suggesting the problem, supervision, encouragement and sincere help during this investigation.

## CONTENTS

	Page
I. INTRODUCTION	( 1)
II. REVIEW OF LITERATURE	( 4)
II.1. Equations describing yield response to water and nutrients application.	( 4)
II.1.A. Functional models for single variable response curves.	( 5)
II.1.A.a. Mitscherlich equation.	( 6)
b. The logistic function.	( 9)
c. The power function (Cobb-Douglas)	(11)
d. The polynomial.	(11)
II.1.B. Functional models for characterizing response surface.	(15)
II.1.B.a. The Mitscherlich-Baule function.	(16)
b. The Maskell "Resistance" formula	(17)
c. The Cobb-Douglas function.	(18)
d. The polynomial function.	(18)
II.1.C. Determination of the optimum dose.	(26)
II.1.C.a. The most profitable application of a single growth factor.	(26)
b. The most profitable application of two growth factors.	(29)
II.2. Soil moisture and its relation to plant growth.	(31)
II.3. The effect of nitrogen fertilization on the growth and yield of plants.	(42)

IV.4.A.	Max. profit expressed in terms of the linearizable quadratic response.	( 83)
IV.4.B.	Max. profit expressed in terms of the Poissonian response.	( 83)
IV.4.C.	Max. profit expressed in terms of the Gaussian response.	( 84)
IV.5.	Functional models to describe simultaneous response to two growth factors.	( 85)
	V. RESULTS AND DISCUSSION	( 87)
V.1.	Effect of soil moisture levels on the growth and yield of plants.	( 87)
V.2.	Effect of nitrogen supply on the growth and yield of plants.	( 92)
V.3.	The interaction between soil moisture and nitrogen supply on growth and yield of plants	( 95)
V.4.	The effect of pot size on yield and response curve.	( 97)
V.5.	Experimental results used to ratify the formulae of linearizable quadratic, Poissonian and Gaussian.	(100)
V.6.	Conventional reference equation.	(119)
V.6.A.	The Mitscherlich's equation.	(119)
V.6.B.	The Cobb-Douglas equation.	(120)
V.6.C.	The logistic function.	(126)
V.7.	Assessment of fertilizer and water requirements for maximum yield.	(130)

V.8.	Determination of the most profitable application of growth factor.	(135)
V.9.	Functional models to describe simultaneous responses to two growth factors	(139)
V.10.	Consumptive use of water as affected by nitrogen and soil moisture levels.	(150)
VI.	Summary and conclusion..	(157)
VII.	References.	(161)
	Arabic summary.	

## I. INTRODUCTION.

Within rapidly developing economics there is an increasing demand for better planning for the use of resources for agricultural production.

Planned agricultural development depends upon a variety of resources and conditions including, management and cultural practices.

It is a known fact that the yield of a particular crop is a function of many factors such as water amount, plant type, climatological factors, soil type and condition, available nutrients and biological materials, diseases and pest. Such factors control and limit the rates of yield.

Several investigators have attempted to express the response of plants to water and nutrient application in a quantitative manner.

Perhaps the oldest and probably best known of the response equations was the one proposed by Mitscherlich:

$$y = A(1 - 10^{-bx}), \quad (1)$$

where "y" is the yield corresponding to the application of a growth factor in the amount "x", "A" is the theoretical

maximum magnitude of the yield; and "c" is an index characteristic of the type of crop, growth factor, and other pertinent condition. One of the other early criticisms of Mitscherlich's equation was that no allowance was made for possible yield depression by harmful excesses of any growth enhancing factor. Mitscherlich, after extensive study of his experimental data introduced a modification of the following form to allow for such depression:

$$y = A(1 - 10^{-cx})(10^{-kx^2}), \quad (2)$$

with the index "k" being called the "factor of injury". The best relationship received a prolonged elaboration, but its application as a whole is difficult due to the presence of three characteristic constants (A, c, and k) and the nonlinearity with respect to the "x", even using logarithmic scales. The contradictory effects of the two indices "c" and "k" interfere making it difficult to determine their discrete values experimentally. In this work a trial was made to find out functions that fulfil physical relevance, simplicity in application, and close agreement with response of yield to water and nitrogen application and their interaction particularly

at high rates of application. A few new functions are innovated and critically compared and reviewed with other existing functions.

## II. REVIEW OF LITERATURE.

The equations describing yield response to water and nutrients application are an important point of research which had received a considerable attention from many investigators, and also about the effects of soil moisture and application of nutrients such as nitrogen on the growth and yield of plants. Following is a review classified according to functional relationships.

### II.1. Equations describing yield response to water and nutrients application.

With the increased interest of production, several papers, such as the following, have been concerned with the choice of a proper functional model for the characterization of input - output relationship in plant growth.

Liebig's (1840) in his subsequent papers and books on this subject mentioned that the relationship between yield and nutrient application or any factor affecting crop yield is linear, and may be expressed in the following form:

$$y = x - R \quad (3)$$

where "y" is the yield obtained when "x" is the amount of the

growth constituent present, and "R" is the resistance to growth of yield. But "R" values are not known. He also establish the law of the minimum factor which he stated as "by the deficiency or absence of one necessary constituent, all the others being present, the soil is rendered barren for all those crops to the life of which that one constituent is indispensable".

Mitscherlich (1930) made experiments with plants grown in sand cultures supplied with "excess" of all nutrients except the one under investigation. His expression is commonly known by the descriptive term "law of diminishing returns". Mitscherlich's work, like Liebig's, produced controversy and has both ardent supporters and critics. His function was modified by other workers. There are also many other functional models for response curves or response surfaces. The following is a review of some of them.

#### II.1.A. Functional models for single - variable response curves.

Mason (1956) mentioned that the Mitscherlich function (Eq. 1) expresses quantitatively the statement that the increase of crop produced by unit increment of the lacking fac-

tors is proportional to the decrement from the maximum, one has:

$$\frac{dy}{dx} = c(A - y), \quad (4)$$

where "y" is the yield obtained when "x" is the amount of the constituent present, "A" and "c" are as defined before.

#### II.1.A.a. Mitscherlich equation.

Upon separation of variables, integration, and assuming that  $y = 0$ , when  $x = 0$ , in the above equation:

$$y = A(1 - e^{-c_1x}) \quad (5)$$

This equation resembles Eq. 1 with the replacement of the quantity " $10^{-cx}$ " with " $e^{-c_1x}$ ". Here  $10^{-cx} = e^{-c_1x}$ , or  $c = 2.3c_1$ .

Mitscherlich maintained that the "c" values in his expression were constant for a given nutrient over different crops and growing conditions.

Most of the early controversy about his work centered around his hypothesis concerning the "c" values. The workers, subsequently mentioned as using the Mitscherlich-type equation, have assumed that "c" is a parameter to be estimated from the data.

This function is expressed in another exponential form by Spillman (1933):

$$y = A(1 - R^x),$$

where "R" is a characteristic constant, and the rest of symbols are as defined before.

Mitscherlich's equation is widely applied in the study of the quantitative relationships between plants and fertilizer application, in such works as:

Balba and Bray (1956) used the Mitscherlich equation:

$$y = A(1 - e^{-c_1b - c_2x}), \quad (7)$$

where "y" is the yield obtained when "b" units of nutrient are in the soil and "x" units are added as fertilizer, for studying the relationship between supply of "P" and the "P" content of wheat grain.

The equation obtained is as follows for the conditions of the run experiments

$$y = 0.5834 (1 - e^{-3.296 \times 10^{-2}b - 2.697 \times 10^{-3}x}), \quad (8)$$

where "y" is the percentage of "P" in the grain in the presence of "x" lb of fertilizer  $P_2O_5$  and "b" lb of soil "P".

Balba and Haley (1956) found the result obtained by

applying the modified Mitscherlich equation:

$$y = A(1 - e^{-c(b + x)}) \quad (9)$$

and the Balba-Bray formula:

$$\frac{x}{x + b} \quad (10)$$

compared with experimental data including rates of application of "P" and soil test values were presented. He calculated values of "P" uptake by plants from the soil "P" and the applied superphosphate and found that they are in agreement with the results obtained from radioactive techniques.

Omar, et al (1969) showed that the 1%  $K_2SO_4$  extractable nitrate nitrogen can be successfully used as a reliable method for predicting available nitrogen status of the U.A.R. soils. The following equation was established to determine yields of corn corresponding to soil test values:

$$y = 100(1 - 10^{-0.01b}), \quad (11)$$

For estimating nitrogenous fertilizers requirement for corn, the following equation was also established:

$$y = 100(1 - 10^{-0.01b - 0.0003F}), \quad (12)$$

One of the early criticisms of Mitscherlich's equation (Eq. 5, or 6) was that no allowance was made for possible