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M. Sc. COURSES

STUDIED BY THE AUTHER (OCTOBER 1977 - OCT

(AT AIN SHAMS UNIVERSITY)

(i) Theory of atomic collisions,
3 hours weekly.

(ii) Mechanics of continuous medium,
3 hours weekly.

(iii) Ordinary differential equations,
3 hours weekly.

ON SCATTERING OF THE μ MESONS

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By

HOSSAM IBRAHIM AHMED HASSAN

Math. Dept. Faculty of Science

SUEZ CANAL UNIVERSITY

To

Math. Dept. Women's College

Ain Shams University

Supervised by

Dr. F. A. AYOUB Ass. Prof.

Ain Shams University

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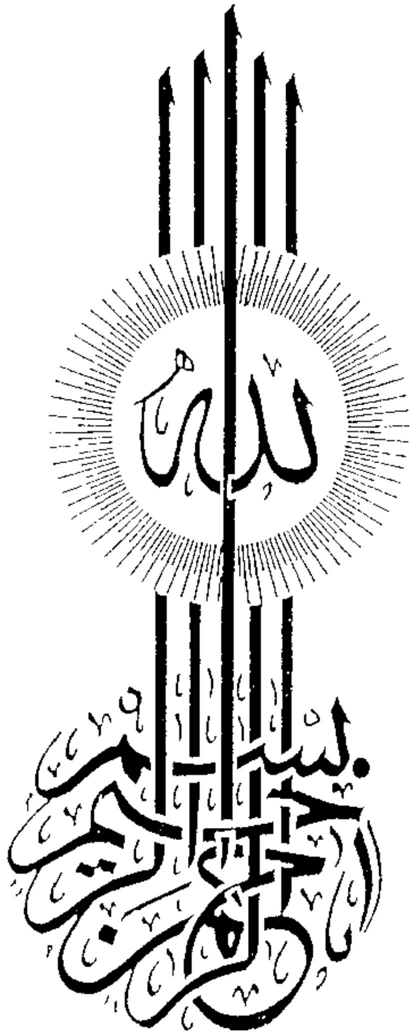
INTRODUCTION


بسم الله الرحمن الرحيم

INTRODUCTION

1- Historical concepts:

Measurements by Rassi in 1932 on the absorption properties of the cosmic radiation showed the existence of two main components, the "hard", or penetrating, component and the "soft", or easily absorbed, component. Coloud chamber experiments soon identified the soft component with electrons but the penetrating particles proved to be more difficult to identify. The possibility of their being protons was soon ruled out by consistancy in the values of their ionisation when observed in cloud chambers. There was an objection to the penetrating particles being electrons and positrons.

All the difficulties disappeared with the postulate that the penetrating particles consisted of particles of mass intermediate between that of the electron and the proton. Laters, the particles termed μ - mesons (muons)

The fluxes of particles of different types of cosmic ray depend on the latitude, their energy, and the conditions of measurement. Very approximately, about 75% of all particles at sea-level are penetrating, and are muons

The decay of pions (π mesons) into muons and neutrinos is the source of most muons in nature. Pions are not

2- Static Properties of the Muon :-

A- Introduction :-

The static properties of the muon - its mass, charge, and magnetic moment - are determined by three independent types of observation, which interact to yield the required parameters.

(a) The magnetic moment experiment : The spin pression frequency of a positive muon f_μ is compared to that of the proton f_p in a magnetic field. This experiment yields the muon g factor :

$$g_\mu = g_p \frac{m_\mu}{m_p} \frac{e_p}{e_\mu}$$

in terms of the proton g factor, ratio of muon to proton mass, and the ratio of proton to muon charge. A chronological tabulation of magnetic moment results is listed in Table 1.

Table - 1 : Magnetic moment determination(muon to proton magnetic ratio).

Date	Reference	Method	Result
2/19[illegible]	Hutchins[illegible]	counters + precession	3.18338 ±.00004
[illegible]/1974	[illegible]	Phase review of previous data	3.[illegible] ±.[illegible]
1/19[illegible]	Casperson	Counters	3.[illegible] ±.0000025
Average	(Error includes scale factor of 1.6)		3.[illegible] ±.0000034

(b) The anomalous moment experiment : The change in polarisation angle (angle between the spin and momentum) of muons stored in a magnetic field B for a time t is a direct measure of the magnetic moment anomaly :

$$\frac{g-2}{2} = \frac{\theta(t) - \theta(0)}{(e/m_{\mu})Bt} \qquad (2)$$

Table - 2 : Chronological tabulation of muon anomalous magnetic moment g_{μ} (units : 10^{-6} $e/2m_{\mu}$)

Date	Reference	Method	Result
5/1969	Bailey	Counters+storage rings.	1165.75 ± .71
11/1975	Bailey	Counters+storage rings.	1165.895 ± 027
11/1977	Bailey	Counters+storage rings.	1165.922 ± 009
Average (Error includes scale factor of 1.0)			1165.9222±0090

(c) [illegible]

which are isotropically distributed. The only explanation of the observed asymmetry is that parity is not conserved in the decay. Thus two results appear from this observation. The muons were longitudinally polarised and parity is not conserved in muon decay.

B- Muon Capture :-

We discuss experiments whose primary goal is the determination of the muon-nuclear weak interaction. The problem has been complicated by practical difficulties. All data are obtained by stopping negative muons in the substance of interest - in all cases, muons are first bound in atomic orbits about the capturing nucleus. To signal a capture event the neutron resulting from μ^- capture is counted, or the recoil of a heavier bound state is detected, or the competition with muon decay is observed as a change in the number and time distribution of electrons.

(1) [illegible] :-

[illegible]

into 1S atomic orbitals around nuclei. If the muon is captured by a proton at rest, the resulting neutron recoils with kinetic energy of about 5 or 6 Mev. In side a nucleus the Pauli principle will for bid such a recoil unless the neutron goes into a state outside the already occupied states. In terms of a Fermi gas model, the recoil momentum is about 100 Mev./C, compared to the radius of the fermic sphere of roughly 200 Mev/C. Some excitation energy($\sim$ 10 or 20 MeV). is imported to the nucleus, but the major part of the muon's rest energy goes to the neutrino.

The capture rate $1 / \tau_c$, is a mode of decay added to the natural decay of the muon. Consequently, the observed decay rate for bounded negative muon is :

$$1/\tau = 1/\tau_c + 1/\tau_\mu$$

where τ_μ is the muon lifetime of the free muon.

(2) <u>Muon capture in complex nuclei :</u>

Since the pioneer work of Wheeler, there have been several [illegible]

[illegible]

CHAPTER (I)

(CHAPTER I)

REARRANGEMENT COLLISIONS

1- Collisions between Massive Particles :-

By massive particles we mean particles of mass large compared with that of an electron, such as atoms, α - particles, etc.

1.1. Slow collisions involving mesons and strange particles

A wide range of new collision phenomena has been exposed through the discovery of the unstable particles pions, muons, K-mesons, and hyperons. Although these particles are all ephemeral they have mean lifetimes (ranging from 10^{-6} to 10^{-10} sec.) long enough to make so many encounters with the atoms of the matter through which they pass that their energies are reduced to nearly thermal values before they decay. The last stages in the life of a strange particle may involve many reactions which are on the form of one of two types: The passage of fast massive particles through matter and chemical reactions. From the point of view of collision theory there are interesting differences however. Thus we have to deal with positive as well as negatively charged particles these [illegible] are [illegible]

[illegible]

one to obtain information about reaction rates in such cases it is clear that a new challenge on a wide front is present to atomic collision theory.

1.2. <u>Slow collisions involving μ -mesons :-</u>

Considerable interest attaches to a knowledge of cross - sections for certain collision processes involving μ -mesons or muons.

In this respect a positive muon is essentially similar in its behaviour to a proton, but the fact that its mass is nine times smaller can be expected to lead to certain new effects. Thus the proton plays a special part in chemical reactions through the so-called hydrogen bond, which owes its importance to the relatively small mass of the proton. We can expect that muons will be considerably more effective in chemical reactions.

2. Maximum cross-section for given angular momentum

In the study of the scattering of a stream of particles by a centre of force, the great majority of collision phenomena, however involving some reaction of the scattered particle on the scatterer. In such case there is a direct energy interchange between the relative translational motion and the internal motion of the colliding systems. There is no transfer of particles between the colliding systems on impact; this however occurs in a number of other types of collision which are called rearrangement collisions.

These collisions include the following :-

1) The capture of electrons from atoms by positively charged particles.

2) Emission of particles from atomic nuclei, with resultant capture of the incident nucleus.

[illegible] and [illegible].

Owing to the complexity of the phenomena even when considering direct collisions only, it is necessary, except in very special cases, to use approximate methods of treatment.

Certain conservation theorems may be derived which are valid under general conditions. They may be used to place limits on the size of the cross-sections and to provide a check on results obtained by approximate methods. We begin by considering these theorems.

We shall consider for simplicity the impact of spinless particles of mass m and speed V with a scattering centre. As usual we resolve the incident wave into partial waves of angular momentum $\{ \ell(\ell+1) \}^{1/2} \hbar$. The new feature which we now include is the possibility of inelastic collisions occuring. However, the asymp totic form of the radial wave function for the particles of energy $\frac{1}{2} m V^2$ can be written as :

$$(kr)^{-1} \quad i^{\ell} \quad (2\ell+1) \sin (kr - \frac{\ell \pi}{2}) + C_{\ell} \quad \frac{e^{ikr}}{r}$$

where $k = mv/\hbar$. The partial elastic scattering cross section is then given by :

$$Q_{\ell}^{el} = \frac{4 \pi}{(2\ell + 1)} \left| C_{\ell} \right|^{2} .$$

Because inelastic collisions may occur the out ward radial flux of particles of speed v will be less than the inward flux by an amount :

$$\frac{v}{r^2} Q_{\ell}^{in} ,$$

the equality only arising when there is no inelastic scattering.

.. Further

$$Q_l^{in} = Q_l^t - Q_l^{el}$$

$$\leqslant Q_l^t - (Q_l^t)^2 / Q_l^{max} .$$

The maximum value of the right - hand side occures when :

$$Q_l^t = \tfrac{1}{2} Q_l^{max} ,$$

so :

$$Q_l^{in} \leqslant \frac{\pi}{k^2} (2l + 1) .$$

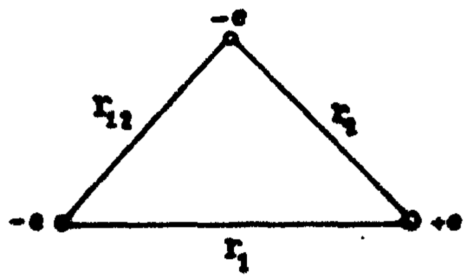
In this case, the equality only arises when Q_l is also equal to

$$\pi (2 l + 1)/k^2 .$$

3. The two state approximation :-

In order to clear the method which must be employed in dealing with inelastic collisions, we first consider the simplest type of collision which occurs in practice, that of electrons with hydrogen atoms. The mass of the electron is small compared with that of the proton, and the motion of the latter in the collision can be neglected We shall assume that the incident and atomic electron are distinguishable.

Figure -1



The wave equation for the system of incident electron and atom is :

$$\left[\frac{\hbar}{2m} \left(\nabla_1^2 + \nabla_2^2 \right) + E + e^2 \left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_{12}} \right) \right] \psi = 0, \qquad (1\text{-}1)$$

r_1 , r_2 and r_{12} are as shown in Figure-1. The energy is :

$$E = \tfrac{1}{2} m v^2 + E_H^o .$$

E_H^o is the energy of the atomic electron in its ground state, v the velocity of the incident electron.

We may expand the function ψ (r_1, r_2) in the form:

$$\psi (r_1 , r_2) = \left(\sum + \int \right) \psi_n (r_2)\, F_n (r_1) . \qquad (1\text{-}2)$$