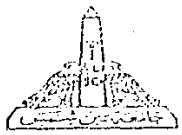

OSCILLATION IN SUBLINEAR DIFFERENTIAL EQUATIONS

THESIS
Submitted In Partial Fulfilment
Of Requirements For The Degree
Of
MASTER OF SCIENCE (M.SC.)
(Pure Mathematics)

BY

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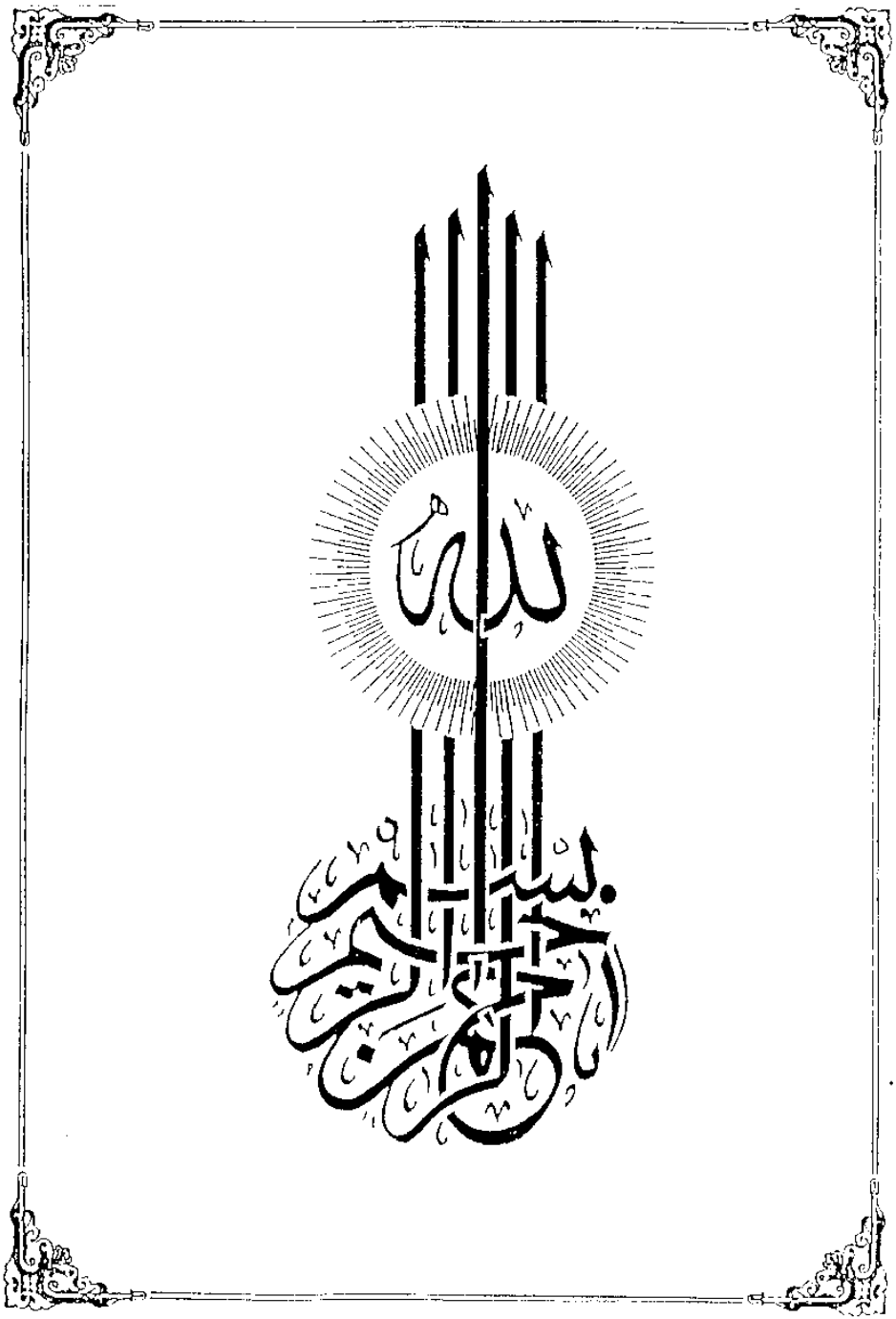

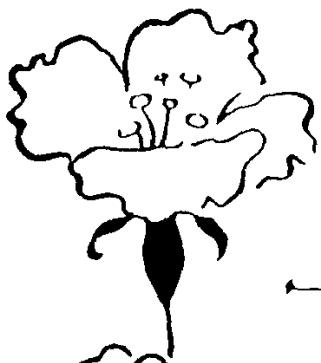

بسم الله الرحمن الرحيم

سبحانك لا علم لنا

الا ما علمتنا

انك أنت العليم

الحكيم

صدق الله العظيم

(سورة البقرة آية ٣٢)

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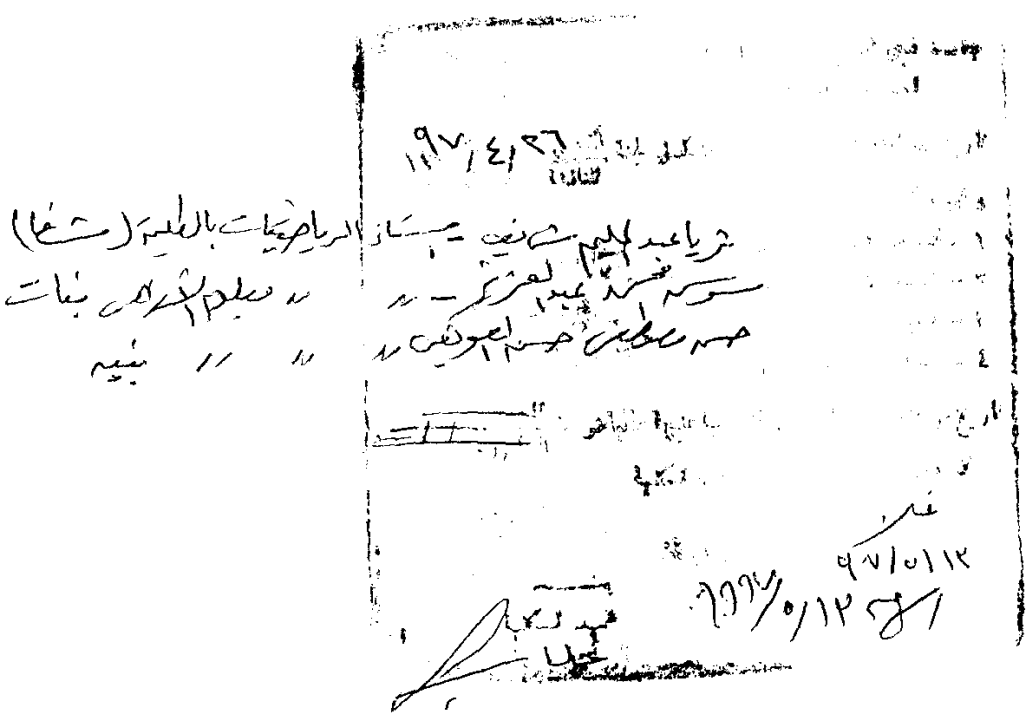
THE STUDENT HAS PASSED THE FOLLOWING COURSES IN PARTIAL FULFILMENT OF REQUIREMENTS FOR M. SC. DEGREE.

1.	**FUNCTIONAL ANALYSIS**	**3h. per week**
2.	**DIFFERENTIAL EQUATIONS**	**3h. per week**
3.	**CALCULUS OF VARIATION**	**3h. per week**



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M. SC. Thesis (Pure Mathematics)

Title of Thesis

OSCILLATION IN SUBLINEAR DIFFERENTIAL EQUATIONS

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ARABIC SUMMARY

ABSTRACT

MONA FATHEY SAYED. Oscillation in sublinear differential equations. Master of Science disertation of pure Mathematics, University College, for Women, Ain Shams University

The main purpose of this disertation is to study the oscillation of a second order sublinear differential equation of the form

$$x'' + q(t)f(x) = 0, \tag{1}$$

where

$$xf(x) > 0, \text{ for all } x \neq 0, \quad \int_0^{\pm\epsilon} \frac{dx}{f(x)} < \infty, \quad \epsilon > 0 \tag{2}$$

and $q(t)$ is called the coefficient. We also considerd a special case of the sublinear equation (1) when $f(x) = |x(t)|^{\nu} sgn\, x(t), \quad 0 < \nu < 1.$

The thesis involves three main parts.
The first one is devoted to sublinear second order ordinary differential equations with nonnegative coefficient.
The second part is conserned with equations where the coefficient may have negative values.
In the third part, we discuss the oscillation and nonoscillation criteria for second order nonlinear differential equations with integrable coefficients.

We introduced two new results concerning the oscillatory solution of an equation of a special type of equation (1)

Key Words:. Sublinear differential equation, strongly sublinear equation, Oscillations, Oscillatory solution, Oscillatory equation, Forced equation.