CONTRIBUTION TO THE ANALYSIS OF SYNCHRONOUS MACHINES

The Effect of Magnetic Saturation on the Transient Performance of Synchronous Machines

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S U M M A R Y

The differential equations for the various currents and voltages in a synchronous machine under transient conditions are given. The variation of the different machine inductances with magnetic saturation is obtained by a new method suggested by the auther. A Computer program has been developed for the solution of these simultaneous differential equations numerically. The program includes also an **iterative** method of successive approximations for systematic selection of the correct values of the machine inductances which are affected by the magnetic saturation. The introduced method gives the actual instantaneous values of the various currents and voltages including all harmonic and non-harmonic oscillations in synchronous machines under transient conditions, given the machine parameters and the magnetization curve.

The values of the alternating current components of the solution are compared with results obtained from another computer program following the semi-graphical method
Rudenberg.

LIST OF PRINCIPAL SYMBOLS

e	instantaneous voltage
i	instantaneous current
θ	pole axis position w,r.t. phase 'a'
L_{aa}	total self inductance of phase 'a'
L_{af}	mutual inductance between phase 'a' and field winding.
L_{ab}	mutual inductance between phases 'a' and 'b'
L_{abo} , L_{aao} & L_{aa2}	constants
R	winding resistance
ψ	flux linkage
ω	angular frequency
t	time
a,b,c	subscripts indicating the armature phases
f	subscript indicating the field winding
d,q	subscripts indicating the direct and quadrature axes
o	subscript indicating zero sequence quantity.
S	saturation factor and a subscript indicating saturation condition.
u	subscript indicating unsaturated condition.
r	subscript indicating the resultant quantity.
l	subscript indicating the leakage.
p	d/ dt
p^{-1}	$\int .dt$
M	the magneto motive force.

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INTRODUCTION

The analysis of the transient characteristics of synchronous machines under sudden short circuit and sudden direct-axis loading have been previously studied by Park[1], Concordia[2], Adkins[3] and many others.

These analysis have been developed under the following assumptions :

i) The relation between the M.M.F. and the flux is linear; the effect of curved magnetic characteristic is neglected.

ii) The armature windings are sinusoidally distributed along the air gap.

iii) The stator slots cause no appreciable variations of any of the rotor inductances with the rotor position.

iv) The angular velocity is constant.

v) The armature and field circuit resistances are considered only in determining the decrement factors for the various components of the solution.

vi) There is no voltage regulating device in the field circuit, i.e. the excitation voltage is held constant.

[illegible] saturated synchronous [illegible] have been previously subjected to much attention. Crary[4], Kingsley[5], Say[6] could include the effect of magnetic saturation in the steady state analysis by choosing an equivalent reactance corresponding to the saturation state of the machine.

However, Rudenberg[7] could introduce a semi-graphical method to determine the R.M.S. value of the fundamental component of the alternating currents under sudden short circuit and sudden direct-axis loading of synchronous machines taking the effect of the curved magnetic characteristic into consideration.

In this semi-graphical method neither the direct current components nor the harmonics generated due to magnetic saturation could be exhibited.

In this thesis a different method of analysis is provided to get the transient characteristics of the synchronous alternators under sudden short circuit and direct-axis loading taking the effect of magnetic saturation into consideration.

CHAPTER I

EXISTING METHODS OF THE ANALYSIS OF SYNCHRONOUS MACHINES

1. Existing methods of the analysis of synchronous machines :

The transient performance of synchronous machines was previously studied by Park[1], Conordia[2], Adkins[3], and many others. All these analysis have been obtained by using the two-reaction theory given by Park[1], (App. A).

1.1. The two-reaction theory

In this theory the three phase windings of the armature are replaced by equivalent fictitious two phase windings having their axes fixed to the poles. One of these fictitious windings has its axis in line with the pole-axis, the direct-axis, the other has its axis in line with the inter-pole axis, the quadrature-axis. These two phases are equivalent to the actual three phases as the currents in both systems produce the same air gap fluxes. The transformation from phase quantities to the direct and quadrature quantities has been obtained on these basis.

In addition to these two quantities zero sequence, quantities are added for unsymmetrical conditions.

The zero sequence quantities are adopted from the analogy with the 'zero sequence component' used in symmetrical component theory.

1.[illegible].1. Transformations

The transformation equations giving the new currents in terms of the actual currents are therefore expressed by the following matrix equations

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120) & \cos(\theta + 120) \\ -\sin\theta & -\sin(\theta - 120) & -\sin(\theta + 120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{1.1}$$

The equations of the inverse transformation, giving the actual currents i_a, i_b, i_c in terms of the new currents i_d, i_q, i_o are obtained by solving the above equations. Thus,

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - 120) & -\sin(\theta - 120) & 1 \\ \cos(\theta + 120) & -\sin(\theta + 120) & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} \tag{1.2}$$

The new voltage are defined by a set of equations exactly similar to those of the currents.

$$
\begin{bmatrix} e_d \\ e_q \\ e_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120) & \cos(\theta + 120) \\ -\sin\theta & -\sin(\theta - 120) & -\sin(\theta + 120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{1.3}
$$

The equations of the inverse transformation are

$$
\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - 120) & -\sin(\theta - 120) & 1 \\ \cos(\theta + 120) & -\sin(\theta + 120) & 1 \end{bmatrix} \begin{bmatrix} e_d \\ e_q \\ e_o \end{bmatrix}
$$

The flux linkage to the equivalent two phase windings in terms of the flux linkage to the three phase windings are defined by a set of equations exactly similar to those of the currents and voltages.

Thus the induced voltage equations can be written in terms of the equivalent fictitious quantities, the direct, quadrature and zero sequence quantities. This has the advantage of simplifying the set of differential equations with variable coefficients to a set of equations with constant coefficients.

The voltage equations in their simplified form are known as Park's equations, (App. A).

These equations are expressed as follows :

$$
\begin{aligned}
e_d &= P\ \psi_d - \psi_q\, P\,\theta - R_a\, i_d \\
e_q &= P\ \psi_q + \psi_d\, P\,\theta - R_a\, i_q \\
e_o &= P\ \psi_o - R_a\, i_o \\
e_f &= P\ \psi_f + R_f\, i_f
\end{aligned}
\qquad (1.5)
$$

At constant rotor speed and linear relation between the M.M.F. and the flux, these are four simultaneous linear differential equations with constant coefficients. The solution of these equations gives the transient performance of the synchronous machine at the specified conditions.

1.1.2. Sudden symmetrical short circuit

For a sudden symmetrical short circuit from no load with an initial field current i_{fo} , the following initial conditions are satisfied.

$i_d = 0.0 \quad , \quad i_q = 0.0$

$P\psi_d = 0.0 \quad , \quad P\psi_q = 0.0$

by substituting these conditions in equations (1.5) thus :

$e_d = 0.0$

$e_q = \psi_d \, P\theta = \psi_d \, \omega = E$

Where E is the amplitude of the rotational voltage at no load corresponding to the excitation current i_{fo}.

The transient quantities of sudden short circuit from these initial conditions are obtained by solving Equations (1.5). Considering the sudden short circuit to be equivalent to applying a voltage having a magnitude equal and opposite to those existing before the short circuit while the field winding is short circuited on itself; i.e. $e_f = 0.0$. The results obtained(3) are the direct, quadrature currents to gether with the induced field current, (App. A).

The corresponding three phase quantities are then obtained by using the inverse of the transformation matrix as given by Equations (1.2). The total field current (i_f) is the sum of the initial value and the induced component (i_{f_i}) ,

$$i_f = i_{f_o} + i_{f_i} \qquad (1.6)$$

1.1.3. Unsymmetrical short circuit

In order to simplify the equations describing the transient characteristics of the machine under unsymmetrical conditions, two other fictitious phase windings $\alpha - \beta$, fixed to the armature side, are used to replace the actual three phase windings.

According to this choice the voltage equations, in terms of α , β quantities, do not include terms depending upon the rotor position θ , because the axes are fixed to the armature just like the actual phases. Therefore, the rotational voltage terms will not be present in the voltage equations. The α-axis is commonly taken along the magnetic axis of phase "a" and the β-axis perpendicular to α-axis in the direction of rotation.

The $\alpha - \beta$ quantities have been defined in terms of d - q quantities as given by Equations (1.7), but they are more directly defined in terms of the original phase quantities by Equations (1.8).

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \qquad (1.7)$$