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# SUDDEN INTRODUCTION OF A MAGNETIC DIPOLE IN A SEMI-INFINITE FLUID

THESIS

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# SUMMARY

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SUMMARY

The problem of magneto-hydrodynamic disturbances in a fluid of finite electrical conductivity has been first discussed by P.H. Roberts and M.G.S. El Mohandis.

In 1957 P.H. Roberts(1) studied the equations governing the propagation of Alven waves in an unbounded fluid of finite electrical conductivity in three dimensions due to the sudden introduction of an infinitesimal current element.

$$\underline{J} = A\delta(t)\delta(\underline{r})$$

In 1959 M.G.S. El Mohandis(2) has extended the problem by discussing the hydromagnetic disturbance and fluid motion due to the sudden introduction of a magnetic dipole in an unbounded fluid where an excitation field $\underline{H}_o$ is prevailing there. The physical importance of Mohandis work has been discussed by P.H. Roberts and R. Hide(3).

In this work the problem is discussed in four chapters, Chapter (I) contains an introduction to the problem. In Chapter (II) we get the basic equations of the problem, in Chapter (III) we solve the problem in an infinite medium, and in Chapter (IV) the problem is solved in the presence of a plane boundary.

The problem in this work is discussed where a magnetic dipole of moment M is introduced with its axis always taken in the z-direction parallel to the plane interface between conducting fluid and insulator.


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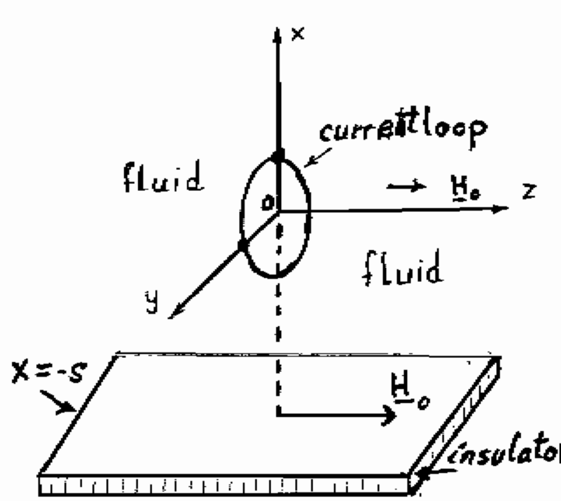
The dipole is replaced by a small current loop, in the (x,y)-plane of strength I, radius a, with its centre always fixed at the origin of coordinates and such that

$$\underline{M} = \lim_{a \to o} \pi a^2 \underline{I}$$

$$\underline{I} = \underline{I}_\phi \, I \delta(\rho - a) H(t)$$

where $\underline{I}_\phi$ is a unit vector in the direction of $\phi$ increasing, $(\rho, \phi, z)$ are cylindrical polar coordinates, H(t) is the Heaviside unit function and $\delta$ is Dirac's delta function.

The direction of the excitation field $\underline{H}_o$ is taken in the direction of the z-axis parallel to the plane boundary separating fluid and insulator and also to the dipole axis (see Fig.) . The small disturbances are treated as a perturbation $\underline{h}$ of $\underline{H}_o$. Mathematical representations are obtained for $\underline{h}$, the magnetic disturbances both in the fluid and in the insulator as well as of $\underline{U}$, the fluid motion. It is worth to note here that the obtained mathematical results have been all expressed in known functions and in such a form that they can be easily computed to obtain necessary figures representing components of $\underline{h}$ and $\underline{U}$ at different intervals of time.

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CHAPTER

I

# INTRODUCTION

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CHAPTER (I)

## INTRODUCTION

The idea that the Earth is a great magnet of permanently magnetized material is now completely disproved, since there is no possibility for the existence of appreciable permanent magnetism at high temperatures which must exist throughout the main body of the Earth. Further, such a theory could not account for either the predominance of the axial main magnetic field of the Earth or its secular variations which is known of its spatial character and its variation with time.

It is therefore believed that the field is generated by electric currents circulating somewhere within the Earth's core of a density which indicates that it may be composed of molten iron, so it would be a good electrical conductor and would therefore provide a suitable medium for the flow of currents generating the disturbed field. It is thus natural to expect that changes in the fluid motion of the core will produce changes in the field which should show time dependences similar to those of the fluid motions producing them. The rapid secular variations of the Earth's magnetic field have therefore been interpreted as indicating the existence of such fluid motions within the core.

The aim of this work is to put a new mathematical theorem which could give an explanation for the existence

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of the permanent Earth's main field as well as for its secular variations with their spatial character and variations with time.

It is supposed that initially we have at rest a semi-infinite, non-viscous, incompressible fluid of hydrostatic pressure P, where a uniform field $\underline{H}_o$ is prevailing through it. A magnetic dipole of moment M is supposed to be suddenly introduced in the fluid at zero time to act as a source of disturbance to the original configuration of the system. By doing so, a small velocity $\underline{U}$ is produced in a certain volume of the fluid and the magnetic field becomes

$$\underline{H} = \underline{H}_o + \underline{h} \quad , \qquad \underline{h} << \underline{H}_o \qquad (1)$$

where

$$\text{div } \underline{h} = 0 \qquad (2)$$

Since we have an incompressible fluid, the equation of continuity shows that

$$\text{div } \underline{U} = 0 \qquad (3)$$

The dipole is considered to be fixed at the origin of coordinates with its axis always directed along the z-axis. The dipole can thus be replaced by a small fluid current loop of strength $\underline{I}$, radius a, always placed in the (x,y)-plane with its centre fixed at the origin of coordinates, such that

$$\underline{M} = \lim_{a \to o} \pi a^2 \underline{I} \qquad (4)$$

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$$\underline{I} = \underline{I}_\phi \; I\delta(\rho - a)H(t) \tag{5}$$

where $\underline{I}_\phi$ is a unit vector in the direction of $\phi$ increasing, $(\rho,\phi,z)$ are cylindrical polar coordinates; $\delta$ is a Dirac's delta and $H(t)$ is the Heaviside unit function.

The fluid is bounded by an insulator with its rigid interface parallel to the dipole axis and at a distance S from the centre of the dipole. The direction of the excitation field $\underline{H}_o$ may be taken to be either parallel or perpendicular to the plane boundary. By considering this together with the different positions of the dipole axis with respect to the direction of $\underline{H}_o$ and the plane boundary, we see that five independent different cases can be obtained as represented in the following figures.

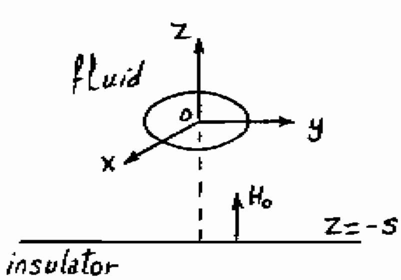

Fig (1)

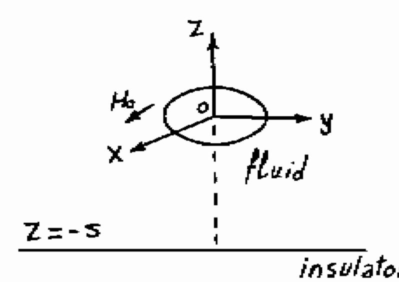

Fig (2)

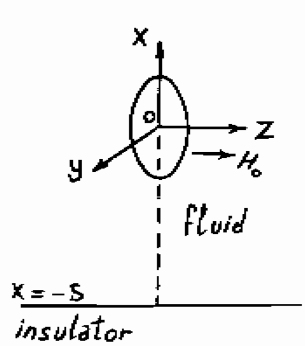

Fig (3)

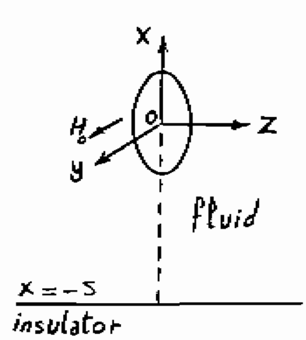

Fig (4)

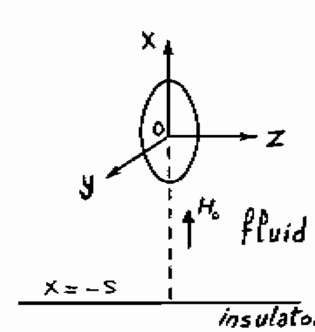

Fig (5)

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The symmetric case as shown in Fig. (1) has been dealt by Mrs. Sayeda Abd El Aziz[4] (1980). The case in Fig. (2) is now under consideration by Mrs. Fahima Camal El Din.

The problem we are going to consider in this work is one of the unsymmetrical cases and is shown in Fig. (3), where the dipole axis is parallel to the plane interface $x = -s$ between fluid and insulator. The magnetic field $\underline{H}_o$ is in the direction of the z-axis parallel to the axis of the dipole and to the plane $x = -s$, so that

$$\underline{H}_o = H_o \underline{I}_z \qquad (6)$$

Mathematical solutions are obtained first in an infinite medium for both the fluid motion $\underline{U}$ and the disturbed field $\underline{h}$. In the case of a bounded medium, mathematical solutions are obtained for the fluid motion $\underline{U}$ as well as for the disturbed field $\underline{h}$ in the fluid, at the plane boundary $x = -s$ and in the insulator.

This work is a modification to Mrs. F. Metwally[5] where results were expressed as a summation in series. In the modified method a Green's function has been obtained. It has also a great power in simplifying calculations of our results since they have been expressed in known functions and not in series.

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CHAPTER

II

# BASIC EQUATION

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# CHAPTER (II)

## BASIC EQUATIONS

### § 2-1 Maxwell's Equations and Hydrodynamic Equations

The phenomena can be described by using Maxwell's equations and the Eulerian hydrodynamic equation:

(A) Maxwell's equations in the presence of a moving electrically conducting matter expressed in electromagnetic units are given by:

$$\text{Curl } \underline{E} = -\frac{\partial \underline{H}}{\partial t} \tag{7}$$

$$\text{Curl } \underline{H} = 4\pi \underline{J} \tag{8}$$

$$\underline{J} = \sigma(\underline{E} + \underline{U} \wedge \underline{H}) \tag{9}$$

where the electric field $\underline{E}$, the electric current $\underline{J}$, the magnetic field $\underline{H}$ and the fluid velocity $\underline{U}$ are each a solenoidal vector.

Taking the curl of equations (8) and (9), substituting in equation (7), we see that

$$\frac{\partial \underline{H}}{\partial t} = \frac{1}{4\pi\sigma} \nabla^2 \underline{H} + \text{Curl}(\underline{U} \wedge \underline{H}) \tag{10}$$

(B) Since we are dealing with an incompressible fluid, with density $\rho$ and hydrostatic pressure P, the Eulerian hydrodynamic equation is

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