APPROXIMATE METHODS FOR SLOW COLLISIONS OF μ MESONS WITH MOLECULES

THESIS

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By

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S U M M A R Y

The present thesis consists of an introduction and three chapters.

The introduction contains the elementary properties of μ^+ mesons and a short paragraph about the elementary particles in quantum mechanics.

The first chapter contains the rearrangement collisions and the reaction of μ mesons with hydrogen molecules.

The second chapter contains the basic wave functions ψ, ϕ, F, g.

The wave equations of the last two functions has been solved numerically for excited states of the two molecules. This chapter contains also the calculation of the cross-sections for excited vibrational and rotational states for the foregoing reaction.

In the third chapter, the calculations of the cross- sections in terms of the phase shifts will take aplace.

In The later has been calculated using Shwinger's method for $\ell = 0$, $n = 0, 1, 2, 3$.

It is found that the calculated values are comparable with the mentioned above in the forgoing chapter and with the theoretical maximum ones.

CONTENTS

INTRODUCTION

MESONS:

We know that cosmic radiation is complex in composition and that it is common practic to divide cosomic rays into two components, the soft component and the hard component.[31] The soft component of cosmic rays is absorbed comparatively readily by matter, from 5 to 10 cm. of lead being enough to absorb it entirely. The hard component, on the contrary, is very slightly absorbed by matter. A perceptible part of this radiation passes through layers of lead a metre and more in thickness. Recall that the complex nature of Becquerel radition was discolesed due to the different character of absorption of alpha, beta and gamma rays that comprise it. However, after the experiments of both. Kothorster and Skobeltsyn, osmic rays were regarded as a stream of electrons and it was though that the nature of the two components is the same. It is true that Anderson's discovery modified these views somewhat. It turned out that positrons are also found in the cosmic radiation registered at the earth's surface. Insofar as it was believed that the hard and soft components of cosmic radiation are of the same type, their different absorption in matter had to be explained by differences in their energies. To explain the capacity of cosmic electrons to traverse big thickness of matter, it was assumed that these electrons (and positrons) possess stupendous kinetic energies. The soft components of cosmic radiation is a flow of electrons and positrons, whose kinetic energy reaches a value of the order of 100 million electron-volts, while the hard component consists of electrons and positrons with energies upwards of one thousand -million electron volts.

The mass of mesons:

An interesting method of determining the mass of mesons was developed by a group of soviet scientists Headed by A.I. Alikhanov and A.I. Alikhanyan. This group of workers organized a permanent station for cosmic radiation studies on Mt. Alagoyz at an altitude of 3,250 m.Work at this station is still in progress. The magnet poles were rectangular in cross-section with sides of 50 cm x 12 cm. The magnet was set up with the 50 cm. side of the rectangle standing vertical. This was done so that the incoming cosmic particles would cover the greatest possible distance in the magnetic field. This magnet was used to study the deflection of mesons in a magnetic field. The magnitude of the deflection made it possible to determine the meson mass.

The Lifetime of a meson:

The discovery of the meson was an outstanding even that extended the list of elementary particles. It was also found that mesons play an important part in the interaction of nuclear particles. This was the reson why an intensive study of the newly discovered particles began. Already the first observation with cloud chambers placed in magnetic fields showed that there are mesons charged positively and negatively. The magnitude of the meson charge has not been measured directly, from indirect data, however, it was found to be equal to that of an electron. Mesons turned out to be unstable particles. They live only a short time and then decay. The decay time of a meson depends on its energy. When a meson decays, the energy associated with its mass is converted into the kinetic energy of other, lighter particles that arise from the decay. This is what underlies the method of measuring the half-lives of mesons. To make the essence of this method clear, the reader's attention is directed to a procedure used at present to determine the direction of motion of high-energy ionizing particles.

Heavy Mesons:-

The investigation of powell and occhielini, Alikhanov, and Alikhanyan, and many others have show that in addition to the pi- and mu-mesons, there exist other, heavier mesons and also particles of mass intermediate between that of a proton and a deuteron. All these particles are unstable. They decay after a very short lifetime into other familiar particles such as pi-and mu-mesons, electrons and positrons. At present, we may regard and establish the existence of mesons possessing the following values of mass:

a- Mesons of massclose to 500 m. The mass of these mesons has not been established very accurately. These mesons are nuclear in active just as mu-mesons. Among the particles at originate in nuclear explosions, mesons of mass 500 m are not observed. They are apparently the decay product of heavier mesons.

b) Theta- mesons (θ^0 and $\theta^\pm$ mesons). These are neutral and charged mesons of mass clos to 800 m. that decay according to the schemes :

$$\theta^0 \longrightarrow \pi^+ + \pi^-$$
$$\theta^+ \longrightarrow \pi^\pm + \pi^0$$

c) K-mesons are charged mesons of mass close to 1,25 m_o that decay according to the scheme:

$$K^\pm \longrightarrow \pi^\pm + \theta^0$$

d) Tau-mesons (τ-mesons) have a mass of 970 m_e. Tau-mesons have been encountered with positive and negative charges, they decay into three light mesons:

$$\tau^+ \longrightarrow \pi^\pm + \pi^+ + \pi^-$$
$$\tau^\pm \longrightarrow \pi^+ + \pi^0 + \pi^0$$

e) Kappa-mesons, (χ-mesons) have an approximate mass of 1,250m_o. A parabolic transformation scheme for Kappa-mesons is the following.

$$\chi^\pm \longrightarrow \mu^\pm + 2 \text{ neutral particles}$$

Classification of elementary particles:

All matter around us is made of elementary particles. All known processes and interactions in nature are due to the interaction between elementary particles. The present known elementary particles can be divided into four classes. The first class contains only one particle- the photon The second consists of leplons: electron, muon, nuetrino, and their antiparticles. The third one comprises the mesons: three π-mesons and four -K-mesons. The fourth one contains baryons and antibaryons. All these particles are enumerated in table (1). Besidesthe mesons and baryans enumerated in table 1 other particles are known which are not included in the table because of their extremely short lifetimes. These "particles" live for such a short time that they manifest themselves only in the form of resonances in reactions at high energies.

TYPES OF INTERACTIONS

There are four types of elementary -particle interactions, sharply differing from one another: the gravitational, the electromagnetic, the strong and the weak. The gravitational interaction has a very small coupling constant (it is very weak), and, if its character does not change sharply at small distance, its role is insignificant for the phenomena that we are going to consider. Indeed, the energy of gravitational interation of two protons set apart at a distance r is equal to

$$\mu \frac{m^2}{r}$$

where μ is the Newtonian constant, while m is the proton mass.

Class	Particle	Mass (MeV)	Mean life (sec)
Photon	γ	0	∞
Leptons	ν_e	$< 2 \times 10^{-4}$	∞
	ν_μ	< 4	∞
	e	0.511006 ± 0.000002	∞
Mesons	$\pi^\pm$	139.60 ± 0.05	$(2.551 \pm 0.026) \times 10^{-8}$
	π^0	135.01 ± 0.05	$(1.80 \pm 0.29) \times 10^{-16}$
	η^0	548.7 ± 0.5	$\Gamma < 10$ MeV
Baryons	P	938.256 ± 0.005	∞
	n	939.550 ± 0.005	$(1.01 \pm 0.03) \times 10^3$

Table 1 Masses and mean lives of stable elementary particles.

The electromagnetic interaction, i.e. interaction of charged particles with photons and (in consequence of photon exchange) with each other, is characterized by the value of electric charge e. the energy of the coulomb interaction of two protons set apart at a distance r is equal to α/r where $\alpha = e^2 = \frac{1}{137}$, while e is the proton charge. For distances $r \sim m^{-1}$ between particles the energy of electromagnetic interaction amounts to αm and is small in comparsion with the proper energy m of the particles. The relative weakness of electromagnetic interaction was used in constructing quantum electrodynamics, i.e. theory of interaction of electrons with photons. The smallness of the constant α allows one to consider electromagnetic interaction as a small perturbation and to develop in quantum electrodynamics the methods of perturbation theory. The mathematical methods of quantum electrodynamics enable calculations to be carried out with an accuracy exceeding that of contemporary experiments.

The strong interaction, i.e. the interaction between mesons and baryons, in contrast to the gravitional or electromagnetic, is short range. The energy of the strong interaction between two particles at a distance larger than 10^{-13} to 10^{-12} cm is negligible, but at smaller distances. ($r \sim 10^{-14}$ cm) the energy of strong interaction becomes of the same order of magnitude as the mass of the strongly interacting particles. If the photon emission by the electron is characterized by the dimensioneless constant α, the pion emission by the nucleon can be chracterized by the constant $g^2 \simeq 14$.

The weak interaction, responsible, in main, for elementary-particle decays, is very short-range, its effective radius is apparently substantially smaller than that of the strong interaction it amounts to about 10^{-17} cm. The radius of the weak interaction and, consequently, its dependence on the momentum of the interacting particles, are only beginning to be investigated. We shall return to this later on. At low momenta the week interaction can be characterized by the week interaction constant $G = 10^{-5}/m^2$. The corresponding energy of the week interaction of two protons set apart at a distance $1/m$ amounts to about 10^{-5}m. This allows one to consider the week interaction, in a number of cases, as a small perturbation, and to calculate slow processes due to it in the first order of a perturbation theory in the constant G. Are elementary particles actually elementary ? This is quite natural when one takes into account the number of elementary particles that seem to exist at present. If, in addition, the number of extremely unstable excited states is added in, the sum is not lower than that of elements Mendeleyer known. It is therefore understandable why numerous attempts to "Lower" the number of elementary particles have been made. The most radical of these attempts proceed from the assumption that all observed particles and interactions are manifestations of a unique non-linear spinor field whose non-Linearity is charactrized by a constant, in the concrete development of this attractive idea, there are very serious difficulties. Such quantum numbers as electric charge, baryonic charge, strangeness, leptonic charge, leptonic strangeness (which distinguishes the muon from the electron) cannot as yet be obtained as characteristic number of some unique fundamental Lagranian. However, the question need not necessarily be formulated in the form that either all particles (fields) are elementary, or that there is only one fundamental field, One may adopt the standpoint that all elementary particles are characteristic states of a system of several fundamental fields interacting with one another. In this case individual fields act like carriers of a certain set of quantum numbers. This idea underlies numerous composite models of elementary particles. In such an approach it is natural to take as a principle of the theory the requirement that the number of fundamental fields and constants of interaction between then should be minimum.