AIN SHAMS UNIVERSITY

UNIVERSITY COLLEGE FOR WOMEN

DEPARTMENT OF MATHEMATICS

THERMAL STRESS IN LINEAR AND NON HOMOGENEOUS ELASTIC PLATES

THESIS

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A C K N O W L E D G E M E N T

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بسم الله الرحمن الرحيم

S U M M A R Y

The present thesis consists of three chapters:

The first chapter contains the basic concepts and definitions from the theory of thermoelasticity in two dimensions and the plane problem of the thermoelastic bodies the properties of which are temperature-dependent.

The second chapter contains the formulation and the solution of the problem in elastic plate with circular hole having temperature independent modulus of elasticity under the effect of steady state heat flow and submitted to a uniform tension (pressure) of magnitude (P). The solution of the problem is given in (1.2) represented as a sum of solution of two problems, the first is the thermal problem assuming P = 0 and the second is the problem of deformation of the plate under the pressure without temperature and the stresses distribution are found in this case.

The third chapter contains the solution of the nonhomogeneous problem when modulus of elasticity is temperature-dependent.

The solution is obtained by the method of successive approximation.

The fourth chapter contains a numerical example which is computed for :

$$\nu = \frac{1}{3} \quad , \quad E = E_0(1-\beta T) \quad , \quad E_1 = -E_0\beta T$$

The stresses distribution in this case are found and the relations between the stresses and radius is obtain , the relations between the maximum value of circumferential stress and the temperature and that of pressure are obtained and the stress distribution on the circumference of the hole is found.

The effect of the temperature-dependent modulus of elasticity is to decrease the maximum value of the circumferential stresses and by comparing with the case of temperature-independent modulus of elasticity.

For example the decreasing reaches to :

0.11% - 0.33%

C O N T E N T S

I N T R O D U C T I O N

In recent years because of the use of structural elements ot elevated temperature, a trend of investigation in thermoelasticity is created in which the influence of temperature on thermal and mechanical properties of the structure is taken into account.

The operating temperatures of modern aircraft propulsion units are of such magnitudes that the existing classical elastic thermal-stress solutions no longer truly describe actual phenomena.

These solutions ignore the behaviour of material properties under temperature and, consequently, become only approximations at higher temperatures where not only material properties vary appreciably with temperature but in addition Hook's Law is no longer applicable even at very low stresses.

In this problem it is assumed that the material properties are temperature dependent, but HOOK's law is applicable.

CHAPTER (1)

BASIC CONCEPTS AND DEFINITIONS

CHAPTER I

BASIC CONCEPTS AND DEFINITIONS

1.1: The Theory of Thermoelasticity in Two Dimensions:

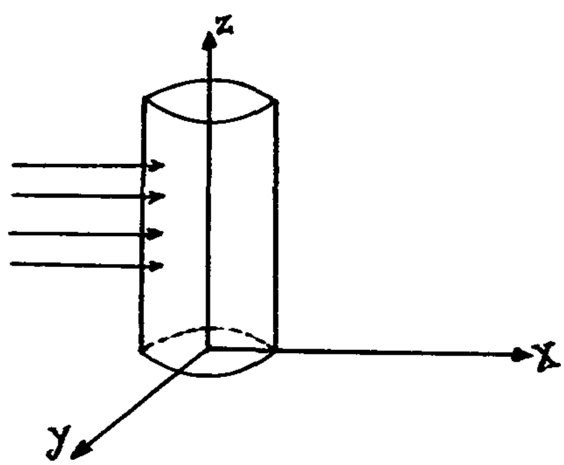
One of the causes of stress in a body is non-uniform heating. With rising temperature the elements of the body expand, such an expansion generally cannot proceed freely in a continuous body, and stresses due to the heating are set up.

We now proceed to a large category of problems of the theory of thermoelasticity which are important for practical application and at the same time admit considerable simplification in the mathematical aspect of solution.

Simplification implies that one may disregard one of the co-ordinate axes in these problems, for instance oz, and consider that the whole phenomenon takes place in one plane ($r\theta$). We encounter something like it in many problems. These problems fall into two groups opposite in a sense but common by the mathematical form of solution.

i) <u>The plane strain problem:</u>

In the plane strain problems, the geometry of the body is essentially that if prismatic cylinder with one dimension (oz) much larger than the others, the loads are uniformly distributed with respect to the large dimension and act perpendicular to it as shown in the figure.



In this conditions the unit elongation ε_{rr}, $\varepsilon_{\theta\theta}$, ε_{zz} and the shearing strain components ($\varepsilon_{r\theta}$, $\varepsilon_{\theta z}$, ε_{rz}) will take the form

$$\varepsilon_{zz} = \varepsilon_{\theta z} = \varepsilon_{rz} = 0 \qquad \varepsilon_{rr} = \Phi_1(r,\theta,T)$$
$$\varepsilon_{\theta\theta} = \Phi_2(r,\theta,T) \qquad \varepsilon_{r\theta} = \Phi_3(r,\theta,T) \qquad (1.1)$$

We suppose W = o (displacement along the axis oz) everywhere, while the other two, (u,v) (radial and circumference displacements) are not dependent on the co-ordinate (z); this case is thus characterised by the following conditions which are valid throughout the body :

$$u = f_1(r,\theta) \quad , \quad v = f_2(r, \theta) \qquad (1.2)$$
$$W = o$$

Equations(1.1) and (1.2) show that all displacements and deformations take place exclusively in the directions parallel to the plane (rθ), the pattern of displacements and strains being the same in all sections of the body parallel to the plane (rθ).

From Duhamel-Neumann relations [1]

$$\epsilon_{ij} - \alpha T \delta_{ij} = \frac{1}{2\mu}\left[\sigma_{ij} - \frac{\lambda}{3\lambda+2\mu}\delta_{ij}\sigma_{KK}\right] \qquad (1.3)$$

where

α : Coefficient of linear thermal expansion.

T : temperature.

λ, μ : Lame's coefficients $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$, $\mu = \frac{E}{2(1+\nu)}$

σ_{ij} : Stress components in cylindrical coordinates.

ε_{ij} : Strain " " " "

δ_{ij} : The Kroneker's symbol (i,j = 1,2,3)

$\sigma_{KK} = \sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}$ (σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} are normal stress components in cylindrical coordinates

We have from equation (1.3)

$$\varepsilon_{zz} - \alpha T = \frac{1}{2\mu}\left[\sigma_{zz} - \frac{\lambda}{3\lambda+2\mu}(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})\right]$$

Substituting by the strain component $\varepsilon_{zz} = o$ in the above equation we get

$$\sigma_{zz} = \frac{1}{2(\lambda+\mu)}\left[\lambda(\sigma_{rr}+\sigma_{\theta\theta}) - 2\mu(3\lambda+2\mu)\alpha T\right] \qquad (1.4)$$

Substituting from equation (1.4) in Duhamel-Neumann relations

(1.3) we get

$$\varepsilon_{rr} - \frac{3\lambda+2\mu}{2(\lambda+\mu)}\alpha T = \frac{\lambda+2\mu}{4\mu(\lambda+\mu)}\left[\sigma_{rr} - \frac{\lambda}{\lambda+2\mu}\sigma_{\theta\theta}\right]$$

$$\varepsilon_{\theta\theta} - \frac{3\lambda+2\mu}{2(\lambda+\mu)}\alpha T = \frac{\lambda+2\mu}{4\mu(\lambda+\mu)}\left[\sigma_{\theta\theta} - \frac{\lambda}{\lambda+2\mu}\sigma_{rr}\right] \qquad (1.5)$$

$$\varepsilon_{r\theta} = \frac{1}{2\mu}\sigma_{r\theta}$$

By solving the first and second equations in equation (1.5) with respect to (σ_{rr}, $\sigma_{\theta\theta}$) we get :

$$\sigma_{rr} = (\lambda+2\mu)\left[\varepsilon_{rr} + \frac{\lambda}{\lambda+2\mu}\,\varepsilon_{\theta\theta} - \frac{3\lambda+2\mu}{\lambda+2\mu}\,\alpha T\right]$$

$$\sigma_{\theta\theta} = (\lambda+2\mu)\left[\varepsilon_{\theta\theta} + \frac{\lambda}{\lambda+2\mu}\,\varepsilon_{rr} - \frac{3\lambda+2\mu}{\lambda+2\mu}\,\alpha T\right]$$

$$\sigma_{r\theta} = 2\mu\,\varepsilon_{r\theta} \qquad (1.6)$$

Equations of equilibrium are :

$$\begin{aligned} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + R &= 0 \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} + S &= 0 \end{aligned} \qquad (1.7)$$

Where 'R' is the body force component per unit volume in the radial direction, (s) is the body circumferential force component per unit volume in direction of θ-increasing.

The surface condition take the form :

$$\begin{aligned} P_{r,n} &= \sigma_{rr}\,Cos(nr) + \sigma_{r\theta}\,Cos(n\theta) \\ P_{\theta,n} &= \sigma_{r\theta}\,Cos(nr) + \sigma_{\theta\theta}\,Cos(n\theta) \end{aligned} \qquad (1.8)$$

Where $P_{r,n}$, $P_{\theta,n}$ are projection on the radial direction and circumference direction of the total stress P_n acting on the area of the plane, (n) is the outer normal to the area of the plane.

The general solution of equation (1.7) contains one arbitrary function $\Phi(r, \theta)$ of independent variable (r, θ) and has a simple form

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \Phi}{\partial r^2} \qquad (1.9)$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial \theta}\right)$$

It is necessary to add to these equations the condition of compatibility of strain components it has in this case the form:

$$\frac{\partial^2 \mathcal{E}_{rr}}{\partial \theta^2} + \frac{\partial^2 \mathcal{E}_{\theta\theta}}{\partial r^2} - \frac{\partial^2 \mathcal{E}_{r\theta}}{\partial r \partial \theta} = 0 \qquad (1.10)$$

From Cauchy relation we have the relation between deformations and displacements in the form:

$$\mathcal{E}_{rr} = \frac{\partial u}{\partial r}, \qquad \mathcal{E}_{\theta\theta} = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}$$

$$\mathcal{E}_{r\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right) \qquad (1.11)$$

Equations (1.5) - (1.11) make it possible to proceed to the solution of the problem of plane strain.