NONLINEAR MULTIPARAMETER PROBLEMS

THESIS
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of
Master of Science in Mathematics

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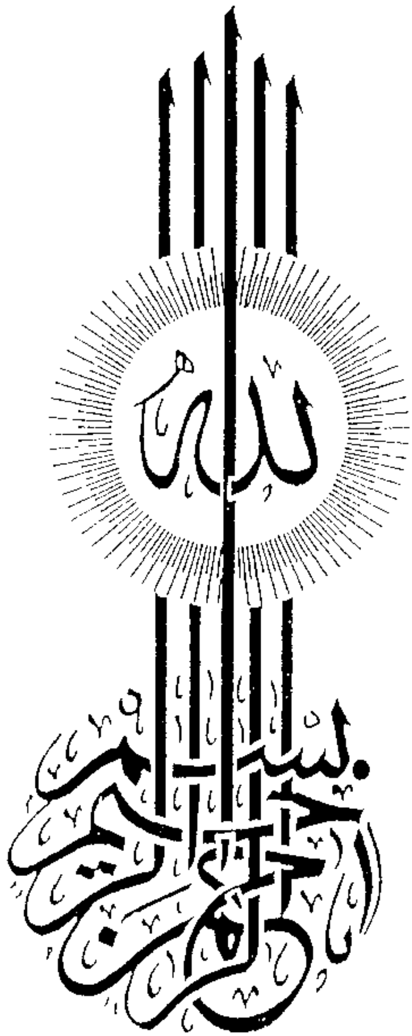
الله

بسم الله الرحمن الرحيم

وَقُلِ اعْمَلُوا فَسَيَرَى اللَّهُ عَمَلَكُمْ وَرَسُولُهُ وَالْمُؤْمِنُونَ

« صدق الله العظيم »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THE STUDENT HAS PASSED THE FOLLOWING COURSES IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE DEGREE OF M. Sc.

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SUMMARY

Consider the problem of vibration of a uniform string stretched between two fixed points. The problem describing this vibration is given by

$$u_{tt}(x,t) = c^2 u_{xx}(x,t),$$

$$u(0,t) = u(L,t) = 0 ,$$

where L is the length of the string and c is constant. To solve this problem we use the method of separation of variables. We set the solution in the form

$$u(x,t) = X(x)\, T(t),$$

this yields to a system of ordinary differential equations of the form

$$X'' + \lambda X = 0,$$

$$T'' + \lambda c^2 T = 0 .$$

This system is the simplest example of one parameter problems. Another example arises when we consider the problem of vibration of a rectangular membrane with a clamped boundary. Application of separation of variables leads to the study of eigenvalue problems for a pair of ordinary differential equations each of them contains the same parameters.

From the above examples we see that one of the main sources of multiparameter problems is the use of the method of separation of variables in the solution of boundary value problems associated with certain partial differential equations. For the many sources of multiparameter problems we refere to [4], [35], [48], [49].

In (1982) Amer [1] investigated nonlinear non-homogeneous multiparameter problems of the form

$$M_k u_k := A_k u_k + \sum_{j=1}^{n} \lambda_j B_{kj} u_k = f_k \in H_k, \; u_k \in H_k, \; k=1,\dots,n, \tag{1}$$

where A_k, $B_{kj} : H_k \to H_k$, $k,j=1,\dots,n$ are certain operators defined on some Hilbert space $H_k (k=1,\dots,n)$ and $\lambda_j (j=1\dots,n)$ are, in general, complex parameters. The basic idea in [1] is to use the theory of monotone operators in Hilbert space to study the system (1). Amer also discussed the solvability conditions for nonlinear nonhomogeneous multiparameter problems in Hilbert space. Then, by using the contraction mapping principle he established a constructive method for solving such problems. We noticed that most of the theory developed by Amer is given in Hilbert space but, this theory fails to analyze the solvability conditions for nonlinear Sturm-Liouville's problem for the ordinary differential equation

$$(-|y'(x)|^{p-2}\,y'(x))' + \sum_{j=1}^{2} \lambda_j\, a_j(x)|y(x)|^{p-2}\,y(x) = f(x),$$

$$x \in [0,1] \qquad (2)$$

where $p \geq 2$, $\lambda_j \in R$, $a_j(x)$, $f(x) \in C[0,1]$, $j=1,2$.

In this thesis we extend the theory developed by Amer in [1] to the Banach space case, then we analyze the solvability conditions of the equations (2). The thesis contains three chapters. In Chapter I we present a historical background to the linear multiparameter problems and paying a special attention to the recent results in the theory of nonlinear multiparameter problems. Also,we introduce the concept of tensor product of Hilbert spaces as well as the tensor product of operators defined on such spaces to reformulate multiparameter problems as a one parameter problem. Our aims and motivation of the following work is also given.

Chapters II is concerned with introducing definitions and basic results from the theory of monotone mappings in a Banach space, then by using these results we discuss the questions of existence and uniqueness of solution for nonlinear, nonhomogeneous multiparameter problems of the form

$$Au + \sum_{j=1}^{n} \lambda_j B_j u = f,$$

where A, $B_j (j=1,\ldots,n) : X \to X^*$ are mappings from a reflexive Banach space X to its dual space X^*, satisfying certain monotonicity conditions, $f \in X^*$ and $\lambda_j (j=1,\ldots,n)$ are real parameters. Application of the developed theory to the nonlinear Sturm-Liouville's problem for ordinary differential equation in L^p space is also given. The study of multiparameter systems of equations has been discussed as a straightforward extension to the case of single multiparameter problem. Finally, we extend our results to the complex Banach space allowing the parameters $\lambda_j (j=1,\dots,n)$ to be complex parameters.

This chapter is published in the second A.M.E. Conference held in the Military Technical Collage in Cairo (1986) [3].

In Chapter III we discuss the questions of existence and uniqueness of solution to the nonlinear, nonhomogeneous multiparameter problems which mentioned in Chapter II when some, or all of the mappings involved are not necessarily defined on the whole of the Banach space, but rather subsets of it. We also introduce the concept of maximal monotone mappings in Banach space together with a subjectivity theorem for these mappings. Then we use these results to establish solvability conditions for nonlinear, nonhomogeneous multiparameter problems. Finally, we investigate multiparameter system of equations by treating each of its equations separately.