PROPAGATION OF HYDROMAGNETIC WAVES IN A SEMI INFINITE FLUID (UNSYMMETRIC CASE)

THESIS

Submitted in Partial Fulfilment of the Requirements for the Award of the Master of Science Degree

BY

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1981

A C K N O W L E D G E M E N T

I would lie to express my great gratitude to Prof. M.G.S. EL-MOHANDIS of Mathematics Department, fro suggesting the problem, kind guidance and encourage ment throughout the preparation of this work.

بسم الله الرحمن الرحيم

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CONTENTS

S U M M A R Y

The problem of magneto-hydrodynamic disturbances in a fluid of finite electrical conductivity has been first discussed by P.H. Roberts and M.G.S. El-Mohandis.

In 1957 P.H. Roberts[1] studied the equations governing the propagation of AlFvèn waves in an unbounded fluid of finite electrical conductivity in three dimensions due to the sudden introduction of an infinitesimal current lement.

$\underline{J} = A\delta\ (t)\ \delta(r)$

In 1959 M.G.S. El-Mohandis[2] has extended the problem by discussing the hydromagnetic disturbances and fluid motion due to the sudden introduction of a magnetic dipole in an unbounded fluid where the excitation field $\underline{H}_0$ is prevailing there.

The physical importance of Mohandis work has been discussed by P.H. Roberts and R. Hide.[3]

In this work the problem is discussed where a magnetic dipole of moment M is introduced in a bounded fluid,with its

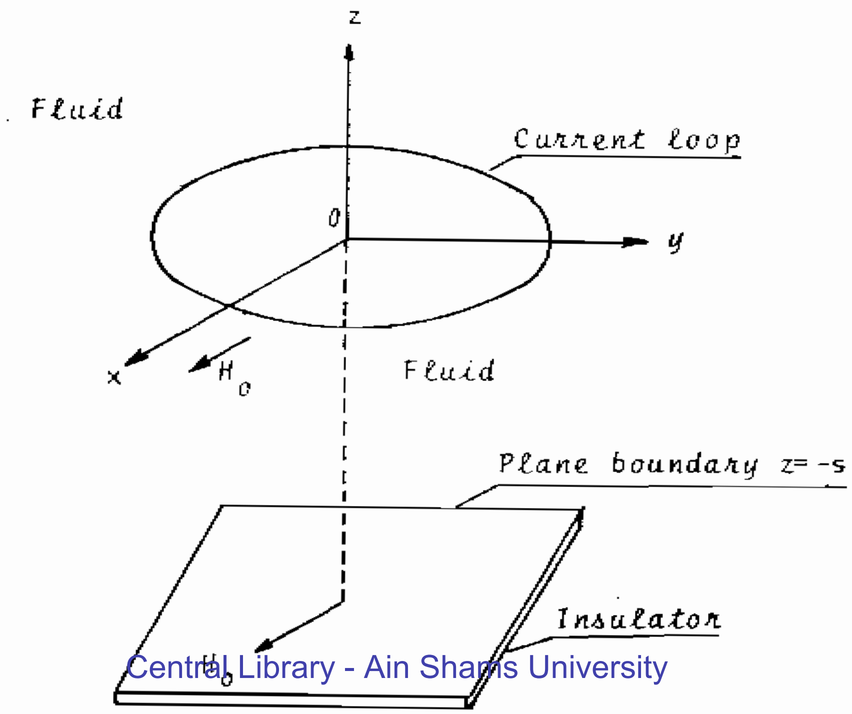
axis always taken in the Z-direction perpendicular to the plane interface between conducting fluid and insulator.

The dipole is replaced by a small current loop, in the (x,y) -plane of strength I, radius a, with centre always fixed at origin of coordinates, and such that

$$M = \lim_{a \to 0} \pi a^2 I$$

$$\underline{I} = \underline{i}_\phi \, I \, \delta(\rho - a) \, H(t)$$

where $\underline{i}_\phi$ is a unit vector in the direction of ϕ increasing (ρ, ϕ, Z) are cylindrical polar coordinates; H(t) is the Heaviside unit function and δ is Dirac's delta function.



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The direction of the excitation field $\underline{H}_o$ is taken in the direction of the x-axis parallel to the plane boundary separating fluid from insulator and perpendicular to the axis of the dipole (See fig.)

This case is one of the four unsymmetrical cases which can be obtained.

The small distrubrances are treated as perturbations $\underline{h}$ of $\underline{H}_o$. Mathematical representations are obtained for $\underline{h}$, the magnetic disturbances both in the fluid and in the insulator as well as of, $\underline{U}$, the fluid motion.

The results thus obtained are epxressed, for the first time, in known functions and not as a summation in series.

CHPATER (I)

I N T R O D U C T I O N

The idea that the Earth is a great magnet of permanently magnetized material is now completely disproved. Since there is no possibility for the existence of appreciable permanent magnetism at high temperatures which much exist throughout the main body of the Earth. Further,such a theory could not account for either the predominance of the axial main magnetic field of the earth or its secular variations which is well known of its spacial character and its variation with time.

It is therefore belived that the field is generated by electric currents circulating somewhere within the Earth's core of a density which indicates that is may be composed of molten iron, so it would be a good electircal conductor and would therefore provide a suitable medium for the flow of currents generating the disturbed field. It is thus natural to expect that changes in the fluid motion of the core will produce changes in the field which should show time depencies similar to those of the fluid motions producing them. The rapid secular variations of the Earth's magnetic filed have therefore been interpretated as implying the existence of such fluid motions within the core.

The aim of this work is to put a new mathematical theorem

Which could give an explanation for the existence of the permane Earth's main field as well as for its secular variations with their spatial character and variations with time.

It is supposed that initially we have at rest a semi-infinite, non- viscous, incompressible fluid of finite electrical conductivity σ, density ρ and hydrostatic pressure p where a uniform field $\underline{H}_o$ is prevailing through it.

A magnetic dipole of moment M is supposed to be suddenly introduced in the fluid at zero time to act as a source of disturbance to the original configuration of the system, by doing so, a small velocity $\underline{u}$ is produced in a certain volume of the fluid and the magnetic field becomes:

$$\left.\begin{array}{l} \underline{H} = \underline{H}_o + \underline{h} \\ \text{where} \\ \underline{h} \ll \underline{H}_o \end{array}\right] \qquad (1)$$

and

$$\operatorname{div} \underline{h} = 0 \qquad (2)$$

since we have an incompressible fluid, the equation of continuity shows that

$$\operatorname{div} \underline{u} = 0 \qquad (3)$$

The dipole is considered to be fixed at the origin, of coordinates with its axis always directed along the Z-axis. The dipole can thus be replaced by a small fluid current loop of strength $\underline{I}$, radius, a always placed in the (x,y) plane with its centre fixed at the origin of coordinates, such that

$$M = \lim_{a \to o} \pi a^2 I \quad \text{------------------------------------} \quad (4)$$

$$\underline{I} = \underline{1}_{\phi} I \delta(\rho - a) H(t) \quad \text{--------------------------} \quad (5)$$

where $\underline{1}_{\phi}$ is a unit vector in the direction of ϕ increasing, (ρ, ϕ, Z) are cylindreical polar coordinates, δ is Dirac's delta function and H(t) is the Heaviside unit function.

The fluid is bounded by an insulator with its rigid interface perpendicular to the dipole axis and at a distance s from the centre of the dipole. The direction of the excitation field $\underline{H}_o$ may be taken to be either parallel or perpendicular to the palne boundary; by considering this together with the different positions of the dipole axis with respect to the direction of $\underline{H}_o$ and the plane boundary, we see that five independent different cases can be obtained as represented in the following figures.

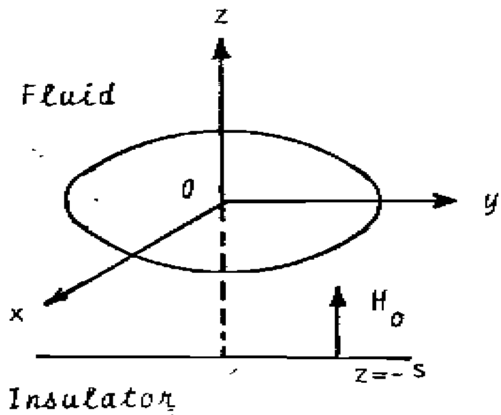

Fig.(1)

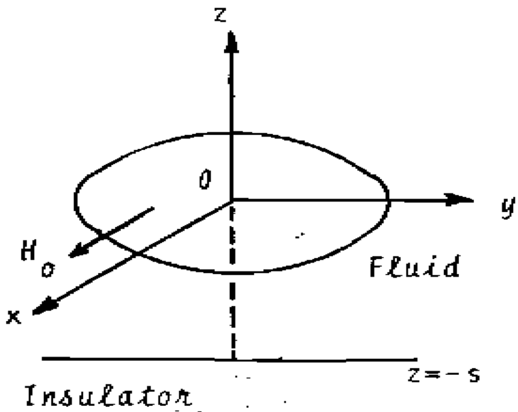

Fig.(2)

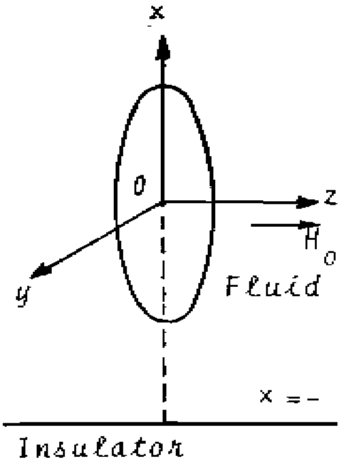

Fig.(3)

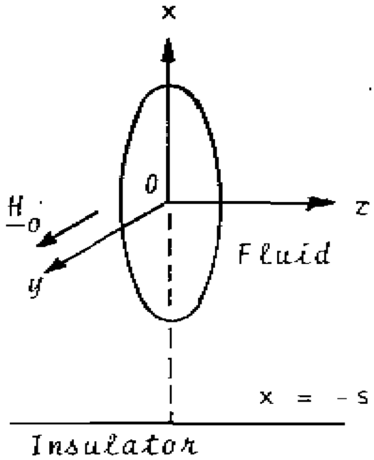

Fig.(4)

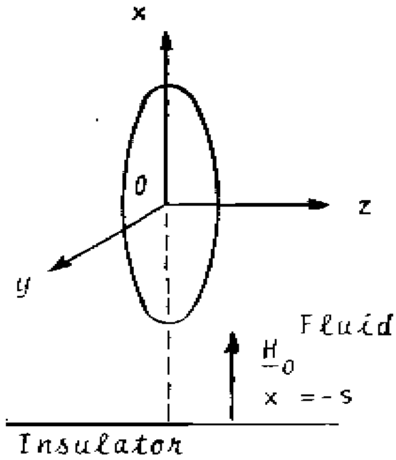

Fig.(5)

The symmetric case as shown in Fig.(1) has been dealth with by Mrs-Sayeda Abd-El-Aziz (1980).[4]

The problem we are going to consider in this work is one of the unsymmetrical cases shown in Fig.(2)where the dipole axis is perpendicular to the plane face of the insulator where

$Z = -s$ and where $\underline{H}_0$ in the direction of the x-axis perpendicular to the axis of the dipole and parallel to the plane interface, such that

$$\underline{H}_o = \underline{H}_o \, \underline{1}_x \quad \text{------------------------------------} \quad (6)$$

CHAPTER II

BASIC EQUATIONS

2.1. Maxwell's Equations and Hydrodynamic equations:

The phenomena can be described by using Maxwell's equations and the Eulerian hydrodynamic equations:

(A) Maxwll's equations in the presence of a moving electrically conducting matter expressed in electromagnetic units are given by:

$$\text{Curl } \underline{E} = - \partial \underline{H}/\partial t \quad \text{------------------------} \quad (7)$$

$$\text{Curl } \underline{H} = 4\pi \underline{J} \quad \text{------------------------------} \quad (8)$$

$$\underline{J} = \sigma \left(\underline{E} + \underline{U} \wedge \underline{H} \right) \quad \text{------------------------------} \quad (9)$$

where the electric field $\underline{E}$,the electric current $\underline{J}$, the magnetic field $\underline{H}$ and the fluid velocity $\underline{U}$ are each a solenoidal vector.

Taking the curl of equations(8) and (9) substituting in equation (7) we see that:

$$\frac{\partial \underline{H}}{\partial t} = \frac{1}{4\pi\sigma} \nabla^2 \underline{H} + \text{curl} \left(\underline{U} \wedge \underline{H} \right) \quad \text{--------------} \quad (10)$$

(B) Since we are dealing with an incompressible fluid, with density ρ and hydrostatic pressure p , the Eulerian hydrodynamic equation is:

$$\frac{d\underline{U}}{dt} = \underline{F} - \frac{1}{\rho} \text{ grad } p \quad (11)$$

where the mobile operator is:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{U} \cdot \text{grad}$$

The only body force $\underline{F}$ in this work is ($\underline{J} \wedge \underline{H}$) which is the mechanical force exerted by magnetic field $\underline{H}$ on an element of fluid carrying a current density $\underline{J}$.

Equation (11), then shows that

$$\frac{\partial \underline{U}}{\partial t} + \underline{U} \cdot \text{grad } \underline{U} = \frac{1}{\rho}(\underline{J} \wedge \underline{H}) - \frac{1}{\rho} \text{ grad } p \quad (12)$$

Using relations(1),(2) ,(3) and (6)neglecting squares and products of the small quantities $\underline{U}$ and $\underline{h}$, we see that:

$$\text{Curl } (\underline{U} \wedge \underline{H}) = \underline{H}_0 \cdot \text{grad } \underline{U}$$

$$= H_0 \frac{\partial \underline{U}}{\partial x} \quad (13)$$

also

$$\underline{J} \wedge \underline{H} = \frac{1}{4\pi} (\text{curl } \underline{H}) \wedge \underline{H}$$

$$= - \frac{\underline{H}_0}{4\pi} \wedge \text{curl } \underline{h}$$

which on using the expansion formula for grad$(\underline{h}.\underline{H}_o)$ shows that:

$$\underline{J}\wedge\underline{H} = -\frac{1}{4\pi}\ \text{grad}(\underline{h}.\underline{H}_o) + \frac{1}{4\pi}\ \underline{H}_o\cdot\text{grad}\ \underline{h}$$

$$= \frac{\underline{H}_o}{4\pi}\ \frac{\partial h}{\partial x} - \frac{1}{8\pi}\text{grad}(\underline{H}_o + \underline{h})^2 \text{ --------------- } (14)$$

Substituting from (13) in (10) and from (14) in (12) ,respectively we see that:

$$\frac{\partial \underline{h}}{\partial t} - \frac{1}{4\pi\sigma}\ \nabla^2\underline{h} = H_o\ \frac{\partial \underline{U}}{\partial x} \text{ --------------------- } (15)$$

and

$$\frac{\partial \underline{U}}{\partial t} = \frac{H_o}{4\pi\rho}\ \frac{\partial \underline{h}}{\partial x} + P \text{ -------------------------- } (16)$$

where

$$\underline{P} = -\ \nabla\ \tilde{\omega} \text{ ------------------------------------ } (17)$$

$$\tilde{\omega} = \frac{1}{\rho}\ [p + (\underline{H}_o + \underline{h})^2] \text{ -------------------------- } (18)$$

Which is the total hydrostatic and magnetic pressure divided by the density.

In an infinite medium,with no signularities,it follows from equations(5),(6) and (16) that we have everywhere

$$\text{div}\ \underline{P} = 0 = -\nabla^2\ \tilde{\omega} \text{ ------------------------- } (19)$$

Equations(15) and (16) together with condition (19) are the required basic equations for hydromagnetic disturbances in an inifinite,inviscid,incompressible fluid.