Ain Shams University
Faculty of Engineering

NUMERICAL SOLUTION OF RETARDED DIFFERENTIAL EQUATIONS

By

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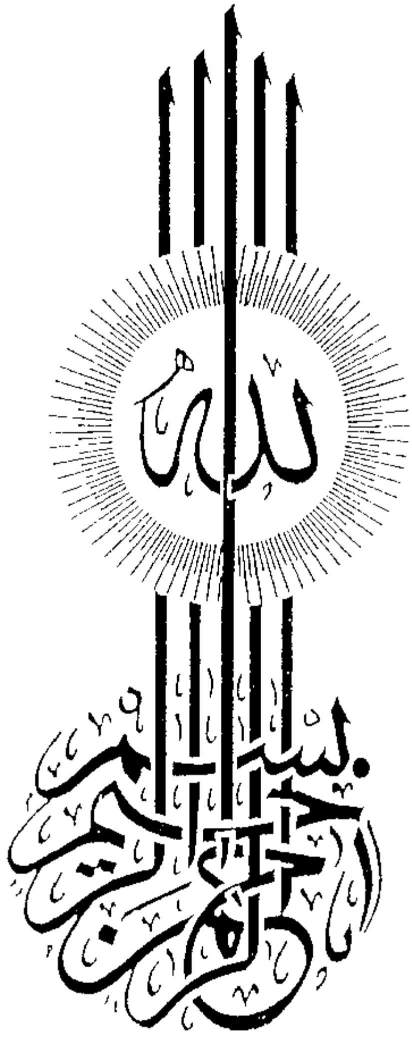
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APPROVAL SHEET

THIS THESIS

ON

NUMERICAL SOLUTION OF RETARDED DIFFERENTIAL EQUATIONS

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STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Master of Science in Engineering Mathematics .

The work included in this thesis was carried out by the author in the Department of Engineering Physics and Mathematics , Ain Shams University , from November 1987 to May 1991 .

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution .

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ABSTRACT

This work aims at finding a numerical solution of the *delay differential equations* (DDEs) defined as those in which the rate of change of the quantity under investigation depends on past and present values of the quantity . For this purpose a modified scheme of *Runge-Kutta* implicit type has been specially developed .

The basic theory of the DDEs , the existence and uniqueness of the solution , the order and stability of the suggested method ; all have been investigated and discussed in detail by means of definitions , proved theorems , various solved examples and well studied cases .

The most important problems arising in the practical use of our proposed method and how to overcome them are extensively discussed in a theoretical manner and numerically tested on four different problems , supplemented by a complete computer program .

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Introduction

1.1 Preliminaries

The fundamental challenges of science are those of description and prediction . Observing certain phenomena, we wish to describe what we see now and how to determine the subsequent behavior .

In many applications , one assumes the system under consideration is governed by a principle of *causality* , that is , the future state of the system is independent of the past states and is determined solely by the present . If it is also assumed that the system is governed by an equation involving the state and rate of change of the state , then , generally , one is considering either ordinary or partial differential equations . Also , as a result of intense and ingenious research , much significant information concerning physical processes can be derived from the analysis of equations of the foregoing simple type . Furthermore , we now possess powerful procedures for obtaining computational solutions using personal computers .

Despite this satisfactory state of affairs as far as differential equations are concerned , we are nevertheless forced to turn to the study of more complex equations . Under closer scrutiny , detailed studies of the real world impel us , albeit reluctantly , to take into account the fact that the principle of *causality* is often only a first approximation to the true situation and the more realistic model would include some of the past states of the system . Also , in some problems it is meaningless not to have dependence on the past .

The theory of such systems is a classification of the so called the *theory of differential equations with deviating arguments* , briefly called *differential-difference equations* .