

*Ain Shams University
Faculty of Engineering*

NUMERICAL SOLUTION OF RETARDED DIFFERENTIAL EQUATIONS

By

ASHRAF ABD EL-HADY AHMED FATHALLAH

B.Sc. June 1984, Electrical Engineering Department,
Communication Section, Ain Shams University

A Thesis

Submitted in Partial Fulfillment of the
Requirements of Degree of
Master of Science
In

ENGINEERING MATHEMATICS

621 38042
A. A.

SUPERVISED BY

Prof. Dr. MOHAMED IBRAHIM HASAN
Professor of Mathematics, Department of
Engineering Physics and Mathematics,
FACULTY OF ENGINEERING
AIN SHAMS UNIVERSITY

Dr. MEDHAT EL-AZAB EL-ZANFALY
Assistant Professor of Mathematics,
MILITARY ACADEMY

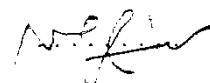
Cairo - 1991

SUPERVISOR COMMITTEE

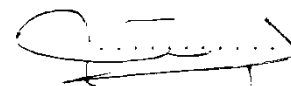
Name , Title and Affiliation

Signature

1. Prof. Dr. Mohamed Ibrahim Hasan
Professor of Mathematics, Department of
Engineering Physics and Mathematics
Faculty of Engineering , Ain Shams University



2. Dr. Medhat El-Azab El-Zanfaly
Assistant Professor of Mathematics
Military Academy



Date : 17/ 4 / 1991



APPROVAL SHEET

THIS THESIS
ON
NUMERICAL SOLUTION OF RETARDED DIFFERENTIAL EQUATIONS
By
ASHRAF ABD EL-HADY AHMED FATHALLAH
HAS BEEN APPROVED BY

Name , Title and Affiliation

Signature

1. Prof. Dr. Hasan Nasr Ismaiel
Professor of Mathematics,
Technology and High Institute, Banha.

Hasan Nasr

2. Prof. Dr. Hasan Mustafa El-Eweidy
Professor of Mathematics,
Faculty of Science, Azhar University.

El-Eweidy

3. Prof. Dr. Mohamed Ibrahim Hasan
Professor of Mathematics, Department of
Engineering Physics and Mathematics
Faculty of Engineering , Ain Shams University.

M. Ibrahim Hasan

4. Dr. Medhat El-Azab El-Zanfaly
Assistant Professor of Mathematics,
Military Academy .

El-Zanfaly

Date : 17/7/1991

Committee in charge

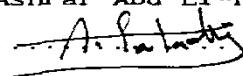
STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Master of Science in Engineering Mathematics .

The work included in this thesis was carried out by the author in the Department of Engineering Physics and Mathematics , Ain Shams University , from November 1987 to May 1991 .

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution .

Name : Ashraf Abd El-Hady Ahmed Fathallah

Signature : 

Date : 17/7/1991

Acknowledgement

I wish to express my cordial gratitude and appreciation to Prof. Dr. Mohamed Ibrahim Hasan , Chairman of Mathematics Department , Faculty of Engineering , Ain Shams University , for his stimulating supervision , constant advice and constructive criticism which made the completion of this work possible .

I am greatly indebted to Dr. Medhat El-Zanfaly , Assistant Professor of Mathematics , Military Academy ,for his diligent and continuous work on reviewing every statement of this work , also for generating many ideas and extensive suggestions for the various chapters which improved the content of this thesis .

First of all , grateful thanks are due to my family , I present my hoped-for success to them .

Thanks are also due to the author's friends and in particular , Eng. Tarek Lasheen and Eng. Wael Fikry for their great help and cooperation in preparing this work .

ABSTRACT

This work aims at finding a numerical solution of the *delay differential equations* (DDEs) defined as those in which the rate of change of the quantity under investigation depends on past and present values of the quantity . For this purpose a modified scheme of *Runge-Kutta* implicit type has been specially developed .

The basic theory of the DDEs , the existence and uniqueness of the solution , the order and stability of the suggested method ; all have been investigated and discussed in detail by means of definitions , proved theorems , various solved examples and well studied cases .

The most important problems arising in the practical use of our proposed method and how to overcome them are extensively discussed in a theoretical manner and numerically tested on four different problems , supplemented by a complete computer program .

Contents

	<i>Page</i>
CHAPTER 1	
INTRODUCTION	1
1.1 Preliminaries	1
1.2 Historical hint	3
1.3 Basic concepts and classifications	3
1.4 Fields of application	8
1.5 A brief description of the order of this work ...	10
CHAPTER 2	
RUNGE-KUTTA INTERPOLATION METHOD	17
2.1 Some generalizations and the basic theory	17
2.1.1 Essential remarks	21
2.2 Runge-kutta interpolation method for retarded equations	24
2.2.1 Basic concepts	24

2.2.2	Supplementary remark	26
2.2.3	The Runge-kutta interpolation formula	27
2.3	Existence and uniqueness theorem for the solution of the RKI method	28
2.4	Stability theory for the RKI method	29
2.4.1	Basic concepts	29
2.4.2	Definitions - Theorem	30
2.4.3	Supplementary remark	37
2.5	Order of the Runge-Kutta Interpolation method	37
2.5.1	Definition	38
2.5.2	Theorems	40
2.5.3	Supplementary remarks	68
2.6	Rigorous determination of the coefficients in the RKI method	69
2.6.1	Example 1	70
2.6.2	Example 2	73
2.6.3	Example 3	77
2.6.4	Supplementary remarks	82
2.7	Case of scalar linear differential equations of constant retard , $C(\alpha)$ -stability	84
2.7.1	Basic theory	84
2.7.2	Definitions	87

2.7.3	Supplementary remarks	91
2.8	Application of the $C(\alpha)$ -stability	92
2.8.1	Case 1	92
2.8.2	Case 2	95
2.8.3	Case 3	98
2.8.4	Case 4	105
CHAPTER 3		
	APPLIED NUMERICAL PROBLEMS	109
3.1	Subject matter	109
3.1.1	Problems under consideration	110
3.2	Applied schemes of the Runge-Kutta Interpolation method	112
3.2.1	RKI-B-1-1 scheme	112
3.2.2	RKI-2-2-1 scheme	112
3.2.3	RKI-3-2-2 scheme	113
3.3	Problems in applying Runge-Kutta Interpolation method	114
3.3.1	Optimal procedure for the RKI method - Definition	116

3.4	Local truncation error (LTE) estimate	119
3.4.1	Definition	119
3.4.2	Application to the considered problems	121
3.4.3	Application to the considered RKI-schemes .	122
3.4.3.1	LTE estimate for RKI-B-1-1 scheme .	122
3.4.3.2	LTE estimate for RKI-2-2-1 scheme .	125
3.4.3.3	LTE estimate for RKI-3-2-2 scheme .	127
3.5	Application of the RKI to the considered problems	129
3.5.1	Numerical results using RKI-B-1-1 scheme ..	130
3.5.2	Numerical results using RKI-2-2-1 scheme ..	133
3.5.3	Numerical results using RKI-3-2-2 scheme ..	137
3.6	Concluding remarks - General observations	139
3.7	Suggestions	142
3.7.1	Empirical formula for the LTE	142
3.7.1.1	Applicability of Empirical Formula	145
3.7.1.2	Application to other problems	148
3.7.2	Supplementary remark	155
3.7.3	Recommended advice	156
3.8	Explanatory figures	157
3.9	Explanatory tables	164
3.9.1	Relative tables for problems 1 and 2	165
3.9.2	Special tables for problem No. 1	169

3.9.3	Special tables for problem No. 2	180
3.9.4	Special tables for deduction of empirical formula	191
3.9.5	Special tables for problem No. 3	194
3.9.6	Special tables for problem No. 4	198
APPENDIX : COMPUTER PROGRAM		202
BIBLIOGRAPHY		280

Chapter 1

Chapter 1

Introduction

1.1 Preliminaries

The fundamental challenges of science are those of description and prediction . Observing certain phenomena, we wish to describe what we see now and how to determine the subsequent behavior .

In many applications , one assumes the system under consideration is governed by a principle of causality , that is , the future state of the system is independent of the past states and is determined solely by the present . If it is also assumed that the system is governed by an equation involving the state and rate of change of the state , then , generally , one is considering either ordinary or partial differential equations . Also , as a result of intense and ingenious research , much significant information concerning physical processes can be derived from the analysis of

equations of the foregoing simple type . Furthermore , we now possess powerful procedures for obtaining computational solutions using personal computers .

Despite this satisfactory state of affairs as far as differential equations are concerned , we are nevertheless forced to turn to the study of more complex equations . Under closer scrutiny , detailed studies of the real world impel us , albeit reluctantly , to take into account the fact that the principle of *causality* is often only a first approximation to the true situation and the more realistic model would include some of the past states of the system . Also , in some problems it is meaningless not to have dependence on the past .

The theory of such systems is a classification of the so called the *theory of differential equations with deviating arguments* , briefly called *differential-difference equations* .