كلية العلوم

ON CLASSES OF CONVEX FUNCTIONS AND THEORY OF ORLICZ SPACES

THESIS
SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS
FOR THE AWARD OF THE (M.Sc.) DEGREE
IN
(Pure Mathematics)

519.2
S. A

BY

Samy Ahmed Ahmad Hareesa

Department of Mathematics
Faculty of Science
Ain Shams University

Under Supervision

Prof. Dr. E. EL-SHOBAKY
Professor of Pure Math. Faculty of Science Ain Shams University

Dr. F. M. GHALEB
Associate Prof. of Pure Math.
Faculty of Science
Ain Shams University

Dr. N. F. MOHAMED
Associate Prof. of Pure Math.
Faculty of Science
Ain Shams University

Submitted at
Faculty of Science
Ain Shams University

1993

Central Library - Ain Shams University

ACKNOWLEDGEMENT

First and for most thanks are to "GOD" the most kind , beneficient and merciful I would like to express my deepest gratitude and thankfulness to Prof. Dr. Entissarat Mohamed El - Shobaky, professor of pure mathematics, Faculty of science, Ain Shams University, for suggesting the interesting point of research, supervision and following the progress of the work with, keen interest, invaluable advice and guidance; and for her help during the preparation of the thesis .

I would like to express my gratitude to Dr. Fayed Mohamed Ghaleb, associate professor of pure mathematics, Faculty of Science, Ain Shams University, and Dr. Nashaat Farid Mohamed, associate professor of pure mathematics, Faculty of Science, Ain Shams University, for their help and kind cooperation .

Finally I wish to thank those whose love, patience, encouragement. and suport have made this endeavour possible my mother, my sons, my sisters : Nabila-Awatef-Naglaa- Manal.

Samy Hareesa

Central Library - Ain Shams University

بسم الله الرحمن الرحيم

Central Library - Ain Shams University

THE M.Sc . COURSES

Studied by the student (1987- 1988)

1. Functional Analysis	2 hours / week
2- Differential Equations	2 hours / week
3- Modern Algebra	2 hours / week
4- Topology	2 hours / week
5- Numerical Analysis [1]	2 hours / week

Central Library - Ain Shams University

CONTENTS

Central Library - Ain Shams University

ABSTRACT

This thesis is to give a survey on some basic facts pertaining to convex functions on normed linear spaces. Special types of convex function, and some topological properties of the Orlicz sequence spaces ℓ_M.

Central Library - Ain Shams University

INTRODUCTION

Central Library - Ain Shams University

INTRODUCTION

In this thesis we study the basic definitions and properties of convex functions on a normed linear space. Two striking examples are subadditive functions and convex functions. Functions which are piecewise convex or concave are basic in calculus, elementary as well as advanced.

Convexity is next to positivety the most fruitful concept in analysis, and its implications are legion.

Convexity is, of course, a geometrical concept, whatever since the days of Minkowski, who was the pioneer and pathfinder in this area, it has become the happy play ground of analyst. The inequality

$$f\left(\frac{1}{2}(x + y)\right) \leq \frac{1}{2}\left(f(x) + f(y)\right)$$

first considered by Otta Hölder in 1889 and in more detail by Danish mathematician and telephone engineer J. L. Jensen (1859-1925) in (1906), characterizes the convex functions.

If $-f$ is convex, then f is said to be concave.

The geometric meaning of the above inequality is that the midpoint of any chord of the curve $t = f(s)$ lies above or on the curve. Actually a convex functions is either continuous or very irregular, unbounded in every interval, thus a measurable convex functions is actually continuous.

Central Library - Ain Shams University

The aim of this thesis is to give a survey on some basic facts pertaining to convex functions on normed linear spaces, special types of convex function, and some topological properties of the Orlicz sequence spaces ℓ_M.

The thesis consists of four chapters:

Chapter 0:

It Contains some defintions and Theorem that will be used.

Chapter I:

In this chapter we study the basic definitions and properties of convex functions on a normed linear space. Such as the continuity of convex functions, differentiable convex functions, ... etc.

Chapter II:

In this chapter we study some types of convex functions such as the N-functions, midconvex functions on normed linear spaces etc and its properties such as superposition of N-functions, complementary of N-functions , etc.

Chapter III:

In this chapter we study some [illegible] of the Orlicz sequence spaces ℓ_M defined with the [illegible] convex function M [illegible] The spaces ℓ_M [illegible] with the classical sequence spaces ℓ_p. The introduction of [illegible] functions has been inspired by obvious rule played by the function t^p in the definition of the spaces ℓ_p or more generally ℓ_M.

Central Library - Ain Shams University

or more generally $\ell_p(M)$, also were replaced t^p by a more general function M and then to consider the set of all sequences of scalars $\{a_n\}$ for which the series $\Sigma M |a_n|$. [17], [18], [21].

Central Library - Ain Shams University

CHAPTER 0

BASIC
DEFINITIONS & THEOREM

Central Library - Ain Shams University

CHAPTER 0

In this chapter we summarize some known definitions and theorem that will be used in this thesis.

Definition 0.1.

A sequence of functions $x_n : S \longrightarrow \mathbb{R}$ is said to be equi-continuous if for all $\varepsilon > 0$ there is $\delta > 0$ such that $d(x, y) < \delta$ implies that $|x_n(x) - x_n(y)| < \varepsilon$ for all n belong to N and δ is independent of n.

Definition 0.2.

A topological space (X, τ) is said to be compact iff for all $\{G_i\}_{i\in I}$ open cover of X there exist $\{G_{i_j}\}_{j=1}^{n}$ finite open subcover of X.

Definition 0.3.

A subset M of a metric space X is said to be totally bounded iff for all $\varepsilon > 0$ there is $\{m_i\}_{i=1}^{n}$ subset of N satisfies the condition $\forall\ y \in M$ there exist $m_{i_0} \in \{m_i\}_{i=1}^{n}$ such that $d(y, m_{i_0}) < \varepsilon$.

Definition 0.4.

A probability space is a totally finite measure space (X, S, μ) for which $\mu(X) = 1$; the measure μ on a probability space is called a probability measure.

Definition 0.5.

Let (X, τ) be a linear topological space then (X, τ) is said to be locally bounded space iff there is a bounded neighbourhood of 0 belong to X.

Central Library - Ain Shams University

Theorem 0.1.

If A is a completely continuous operator mapping, a closed bounded convex set S of Banach space E into itself then there is a point x of S fixed under A such that $A(x)=x$.

Central Library - Ain Shams University

CHAPTER 1

CONVEX FUNCTIONS ON NORMED SPACES

Central Library - Ain Shams University

CHAPTER I

CONVEX FUNCTIONS ON NORMED SPACES

Introduction

In this chapter we give a number of definitions and conventions which we need in our thesis and which form the theoretical base of convex functions. The references to our representation are [1], [4], [5] and [20].

The first section of this chapter deals with basic definitions and results. The second section deals with the continuity of convex functions. The third section is devoted to the differentiable convex functions, and the fourth studies the support of convex functions. Also, the proofs of the basic theorems are given.

1.1. Basic Definitions and Results

Definition 1.1.1.

We say that a set $U \subset L$ (normed linear space) is convex if

$$[\lambda x + (1 - \lambda)y] \in U \qquad \text{where } x, y \in U, \quad \lambda \in [0, 1]$$

Definition 1.1.2.

We merely require that the domain of f be convex i.e. for $x, y \in U$, $\alpha \in [0, 1]$, f will always be defined at $\alpha x + (1 - \alpha)y$.

We then define f to be convex on $U \subset L$ if

Central Library - Ain Shams University