

Some Problems in Recent Kinds of Colouring and Valuation of Graphs

Thesis

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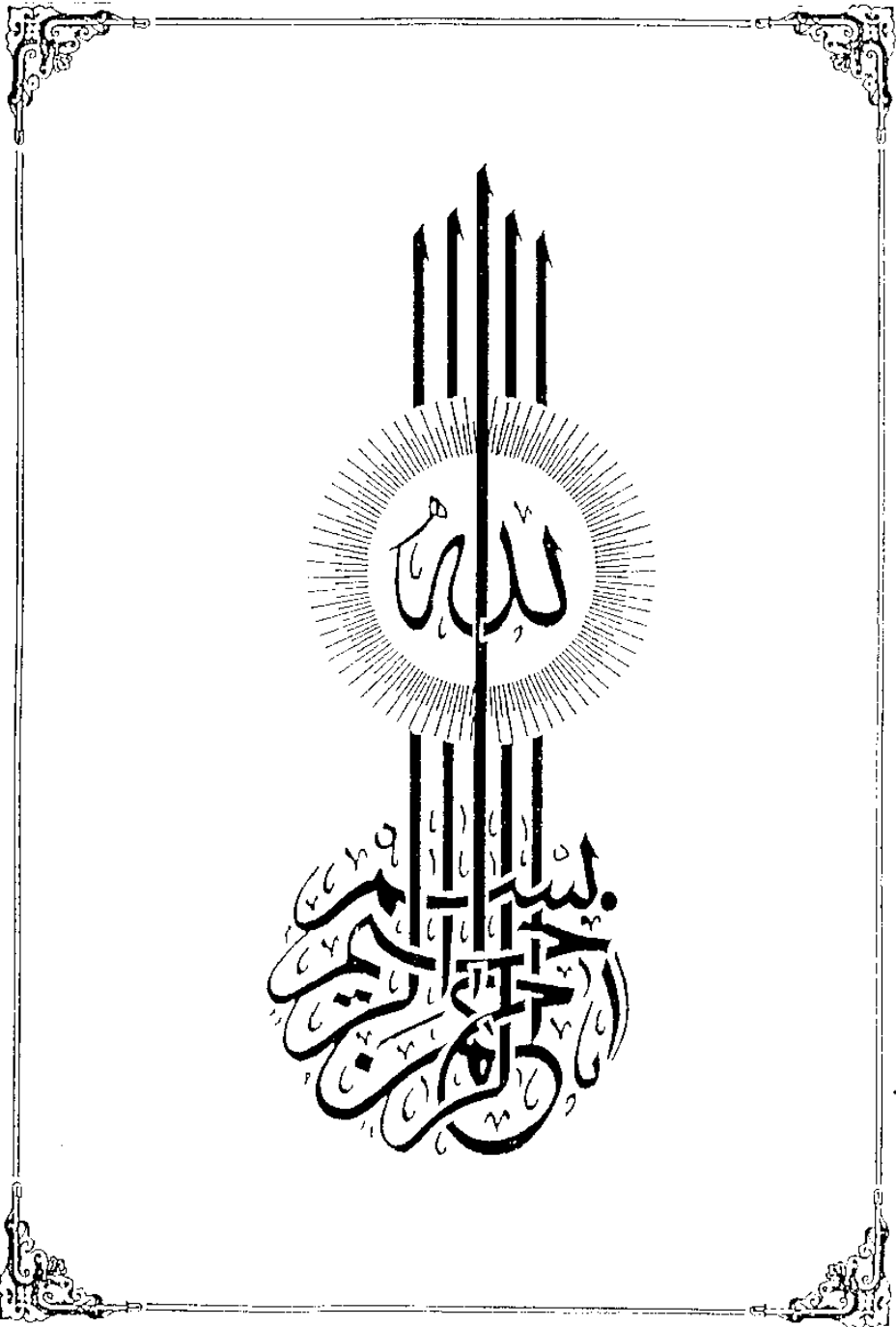
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ARABIC SUMMARY

ABSTRACT

