ON THE FLOW OF MULTICOMPONENT FLUID

THESIS

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بسم الله الرحمن الرحيم

قالوا سبحانك

لا علم لنا إلا ما علمتنا إنك أنت العليم الحكيم

سورة البقرة آية ٣٢ صدق الله العظيم

بسم الله الرحمن الرحيم

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Nomenclature

f_1, f_2, f_3 : Porosity of each phase.

μ_1, μ_2, μ_3 : Dynamic viscosity coefficients of the phases.

$\upsilon_1, \upsilon_2, \upsilon_3$: Kinematic viscosity coefficients of the phases.

k_{12}, k_{13}, k_{23} : Coefficients of the phase interaction.

R : Radius of cylindrical pipe.

Re_1, Re_2, Re_3 : Reynold's number of the first, second, third phase respectively.

P : Pressure

τ_1, τ_2 : Shear stress for each fluid.

ϵ_1, ϵ_2 : Eddy viscosity for each fluid.

k_1, k_2 : Turbulence constants.

h : Half the spacing between plates.

σ : The electrical conductivity of the medium.

μ : The magnetic permeability.

H_o : The external magnetic field.

B_o : The magnetic induction, $B_o = \mu H_o$.

SUMMARY

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The aim of this thesis is to investigate some problems concerning the laminar and turbulent flow of a multiphase electrically conducting and non conducting fluids inside a cylindrical pipe, also between two long plates.

The thesis consists of six chapters :

IN CHAPTER I, we present a literature survey on the subjects involved and their historical developments.

CHAPTER II

Turbulent flow of a mixture consisting of two fluids near a plane wall

This chapter deals with the two component turbulent flow near a plane wall. Equations governing the motion are derived, a new model for the eddy viscosity is suggested and a formula for the velocity distribution is obtained.

The results show that the increase in the phase interaction parameter works to decrease the difference between the velocities of the two fluids.

CHAPTER III

Magnetohydrodynamic turbulent flow near a plane wall for a mixture consisting of two fluids

In this investigation we have used the same model for the eddy viscosity as in the previous chapter, this model gives analytical solution for this problem and predict flow velocity near the wall with satisfactory accuracy. The effects of magnetic field parameters m_1 and m_2, the interaction phase parameter I_2 on the velocity are shown graphically.

IN CHAPTER IV, we have studied two problems:

(1) Motion of an incompressible viscous three-phase media in a circular coaxial annular conduit.

Steady flow of incompressible viscous three-phase media with constant porosities in a circular coaxial annular conduit is considered. The general feature of the flow phase is given when the pressure gradient is negative. Especially, it is shown that the velocity field of the three-phase medium is the same as that of the incompressible Poiseuille flow. The variation of the velocity profiles and flow rate with the various parameters are shown graphically . From the results we see that:

(i) **The velocity at any point of the fluid increases with** increasing the pressure gradient parameter **N** and the maximum velocity nearly occurs in the middle

(ii) **The increasing of the phase interaction parameters between any two of the three components works to** decrease the **absolute value of the difference between** the **velocities of these two fluids.**

(ii) The medium with the lower viscosity has the greater flow rate.

(2) Motion of three-phase media in the space between similar pipes.

We are interested in the flow of three-phase media in the space between similar piping. The behaviour of the velocity profile and the flow rate with the variation of various parameters are shown graphically.

CHAPTER V

Viscous flow of three component media between fixed arcs of concentric circles.

The aim of this problem is to study the effects of the pressure gradient parameter, viscosity and phase interaction parameters on the flow of three component media between fixed arcs of concentric circles which is considered plane parallel. Analytical solutions to the equations governing the velocity and flow rate are derived, and it is found that the velocity and the flow rate increases with increasing of the pressure gradient parameter, and the less viscous of three media moving in combination has the greater velocity, the flow rate increases with increasing of the radius of the outer circle.

CHAPTER VI

Stationary flow of a three-phase media one of the phases is electrically conducting in the presence of transverse magnetic field.

A mathematical description for a flowing mixture of three-phase medium between two parallel planes is developed in the presence of uniform magnetic field when one of the phases is electrically conducting. Specifically, the equations governing the flow of a three-component mixture are derived. These relatively general equations are then reduced to a system of coupled ordinary differential equations describing a steady flow of the mixture between flat plates. The resulting boundary value problem is solved analytically.

The object of this study is to discuss the effects of the magnetic field parameter B_o, the electric field parameter E_o, the phase interaction parameters and the parameter R_E, $R_E = \frac{E_o}{B U_o}$ on the velocity and induced magnetic field profiles. It is found that:

(i) The velocity of the fluid decreases with increasing the magnetic field parameter B_o and increases with increasing of the electric field parameter E_o.

(ii) The induced magnetic field has negative values for $0<z<1$ while for $-1<z<0$ it has a positive values for $R_E \geq 1$, while for $R_E<1$, the induced magnetic field reverses it's direction, and we notice that the induced magnetic field vanishes in the middle of the distance between the two plates.

CHAPTER I

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INTRODUCTION

1.1. Multiphase system

Multiphase flows have become the subject of considerable attention because of their increasing importance in many industrial applications. Flowing mixtures consisting of solid particles entrained in a fluid are relevant to a variety of applications such as pneumatic and hydraulic transport of solid particles and fluidized beds.

A multiphase system consists of mixtures of solid particles, liquid droplets or bubbles in fluids, and may also involve a gas mixture. In a gas mixture, it is understood that the gases are not reacting chemically. Studies of multiphase systems are still going through stages of development including recognition of basic phenomena, development of instrumentation, mathematical representation, and methods of solution.

In a multiphase system a fluid phase or fluid medium contains a particulate phase of any number of chemical components. If the fluid medium is a gas, the particulate phase may consist of gas, solid particles or liquid droplets or both. When the fluid medium is a liquid, the particulate phase may consist of solid particles, gas bubbles, or liquid droplets immisible to the fluid phase.

The dynamic of multiphase systems includes momentum, energy, mass and charge transfers between the phases, whether the process is influenced or not by the presence of a potential field. There are many multiphase systems among engineering equipment and processes. In what follow are few examples:

(i) Gas-solid particle systems: pneumatic conveyors, dust collectors, heterogeneous reactors, metallized propellent rockets.

(ii) Gas-liquid droplet systems: atomizers, dryers, absorbers, combusters, gas cooling, air pollution.

(iii) Liquid-gas bubble systems: absorbers, evaporators, air lift pump, cavitation, flotation.

(iv) Liquid-liquid droplet systems: extraction, homogenizing, sedimentation.

To study the dynamics of multiphase systems researchers have followed two methods of approach:

(i) Treating the dynamics of single particles and then trying to extend it to multi-particle system in an analogous manner as in molecular (kinetic) theory.

(ii) Modifying the continuum mechanics of single-phase fluids in such a way to account for the presence of particulate phase.

The subject of gas-liquid two phase or in general multiphase flows have become increasingly important in numerous pieces of engineering equipment such as process system, heat transfer systems, oil and natural gas pipelines especially nuclear technology such flows are particularly difficult to analyze, except in certain simple conditions

A study of the phenomena occurring in the motion of multiphase media is essential for the solution of many theoretical and engineering problems: the theory of wave propagation in water-saturated ground and dust-saturated air, the theory of fine grinding, the theory of mass entrainment from the surface of a body by a high-parameter flow, the theory of the flow around porous bodies, the theory of the motion of powder gases containing unburned particles, the perfection of water and pneumatic transport, petroleum output and petroleum refining when there is foreign matter in the petroleum, etc. In all these cases the moving material is mixture of two or more media, i.e., phases.

1.2. Differential equations of motion of the multiphase of fluids(Rakhmatolin Equations):

Many investigators are interested in finding the solution for the problem of the motion of two-phase viscous medium such as Rakhmatolin, who has studied and deduced the equations of motion of n-th phase.

i) Rakhmatolin proved that:

"For monochromatic porous medium, the volume and surface, porosity are equal". This theorem is very important, because it helps us calculate the volume and surface forces in the Rakhmatolin equation as in [1].E. G. if the pressure p is acting on an area $d\delta$ then the affecting forces for n-th intermediate media will

$$p\, f_n\, d\delta = \frac{\rho_{nc}}{\rho_{ni}}\, p\, d\delta \quad .$$

Where f_n is the porosity for the n-th phase.

According to the definition of the porosity we have for the n-th media.

$$f_1 + f_2 + \dots\dots\dots f_n = 1 \quad , \tag{1.2.1}$$

$$\rho_{nc} = f_n \rho_{nl} \quad . \tag{1.2.2}$$

(from the theorem of Rakhmatolin) where ρ_{nc} , ρ_{nl}. are the density of the mixture and the real density for the n-th respectively.

From the relations (1.2.1), (1.2.2) we obtain the relation:

$$\frac{\rho_{1c}}{\rho_{11}} + \frac{\rho_{2c}}{\rho_{21}} + \dots\dots\dots + \frac{\rho_{nc}}{\rho_{nl}} = 1 \ . \tag{1.2.3}$$

ii) <u>Equation of continuity (conservation of mass)</u>:

If the mixture consisting from n media then it is obvious from the preceding section, that there are n real densities at each point of the space and n mixture densities, each medium is considered continuous. We notice that each point of the mixture has a number of velocities equal to the number of media.

In this case the law of conservation of mass in Euler's