DEVELOPMENT OF NUCLEAR SHELL MODEL AND ITS APPLICATION TO STUDY THE CLUSTER STRUCTURE OF LIGHT NUCLEI

THESIS

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SUMMARY

An extension of the nuclear one-, two-, three- and four-center shell model in a line to four-center shell model (FCSM) in a plane is presented. Such extension is flexible in the study of the nuclear structure of light nuclei and also the nuclear fission into four fragments. The first part of the present study includes in some detail the analytical solutions for the eigenvalue problems of the previous two-, three- and four-center harmonic oscillator models in a line. The second part, which is the extension of the previous work, gives the solution of the eigenvalue problem for the four center harmonic oscillators not in a line but in a plane both numerically and analytically. As a first step, the problem was treated without $\vec{\ell}.\vec{s}$ and $\vec{\ell}^2$-terms and for mathematical convenience, the symmetrical case with spherical fragments was considered. The single particle energy levels for some nuclei were obtained by calculating the absolute values of the oscillators frequencies. These frequencies were calculated under the condition of volume conservation which imposes the constancy of equipotential surface coinciding with the nuclear surface during the deformation.

The basic results could be summarized as follows:

(1) The relation between the total eigenvalues λ and the dimensionless separation parameter ($\sqrt{\frac{m\omega}{\hbar}}\,a$) was obtained. For no separation ($a = 0$), the eigenvalues take odd integer numbers and the degeneracy is given by $n(n+1)/2$ where n is the order of the level, i.e., 1,3,6,10,15,... while for the asymptotic condition ($a \rightarrow \infty$) the degeneracy is given by $4\times[n(n+1)/2]$, i.e., 4,12,24,40, ...

(2) The oscillator frequencies determined from the constraint of volume conservation yield $\hbar\omega = \frac{41\times1.25}{r(a)}$ [fm.Mev], where $r(a)$ is the cluster radius during deformation.

(3) The single particle energies $E = \frac{\hbar\omega}{2}\cdot\lambda$ were obtained by knowledge of ω and λ. They were plotted against the deformation parameter a for ^{16}O, ^{32}S, ^{48}Cr, ^{64}Ge and ^{208}Pb nuclei. The development of the single oscillator spectrum of the original compound nucleus into a fourfold degenerate oscillator spectrum for the separated fragments was noticed. The resulting single particle schemes describe the starting point, i.e., the nuclear structure of the original spherical nuclei, as well as the final state of the four separated spherical fragments and their individual structure.

(4) The single particle energy schemes showed that the lowest single particle levels are at large deformation already completely degenerate with energies corresponding to the harmonic oscillator states of the fragment nuclei. In the higher states these nuclei still feel the neighbourhood of the brother fragments and therefore show splitting of the level at the fermi surface. This is interpreted in terms of all the core nucleons moving already in the shell model potential of the final nuclei while only the valence nucleons at the fermi surface are responsible for the interaction of the fragments in the final stages of the separation process.

(5) A rough estimation of the binding energies for the four-cluster of ^{16}O, ^{32}S, ^{48}Kr, ^{64}Ge and ^{208}Pb as a function of deformation was obtained by adding the single particle energies up to the fermi level. Each binding energy curve was found to have a minimum at a relative small deformation and a constant value at a separation larger than that corresponds to four touching spheres. These minima were interpreted in terms of a quadruple cluster configuration. The deformation a_{min} and the cluster radius r_{min} were found to approach saturation at large mass number as is physically expected. Also, the minimum binding energies were found to exhibit a linear relationship with the mass number A which could be considered as a quasistability curve for all nuclei forming a symmetrical spherical four-cluster.

INTRODUCTION

The shell model explains important features of nuclear structure. Therefore, many authors have tried to use it also in the discussion of nuclear fission into potential [1,2,3]. However, for nuclear fission and related phenomena it is intuitively evident that the use of two or more centers of force is a more appropriate and powerful tool to describe the realistic situation instead of using one-center potential, e.g. the Nilsson model [4]. The Nilsson model becomes invalid for large deformation in the sense that it does not contain the proper asymptotic behaviour required in the fission process. In particular, the collective potential, which is essentially a sum of single particle energies, will approach infinity for large deformation and, therefore, will not give a fission barrier. One way to overcome this difficulty has been proposed by Strutinsky [5,6] who defines the shell corrections which are to be extracted from the Nilsson model and added into a well-behaved liquid-drop barrier. It should be noted that the liquid drop potential also increases to infinity for large deformations, because of the increasing of the surface energy for needle ellipsoids. This question leads to the development of an asymptotically correct shell model which is based on the double-center potential model [7-13] since the surface of the two center shapes model does not go to infinity for large deformation in contradiction to the case of one-center model. Holzer, Mosel, and Greiner [7] have shown the importance of the two center single particle potential as a starting point for all microscopical fission calculations. The previously developed two-center potential for symmetric fission [7] has been extended to break up into unequal fragments by Maruhn and Greiner [12]. They generalized the model developed in [7] to asymmetric shapes with a smooth joining of the fragments allowing a variable height of the potential barrier between the fragments. A more realistic shell model potential which allows also for continuous states is the cut-off two-center potential was proposed by Ong and Scheid [13]. These authors have calculated the bound states of the two-center oscillator with finite depth.

A further desirable generalization of the model will be the inclusion of shapes containing more than two centers, thus allowing an examination of multiple fission. Thus, an extension of the nuclear two-center oscillator to three- and four-center potentials in a line was investigated by Bergman and Scheefer [14]. The generalization of the shell model based on the harmonic oscillator potential to a system made up of three clusters with spin orbit term and ℓ^2-correction was proposed by Degheidy [15]. The centers of the three clusters may be in arbitrary geometrical configuration in a plane and the clusters may be of different masses. This model extended the work of Diel et al [16] by investigating the behaviour of shell correction for a ternary break up. The results showed that the three center shell model developed by Degheidy [15] is a flexible tool in the study of ternary fission and also in the structure of light nuclei.

The present work, the four-center shell model in a plane, is an extension to the previous work. For mathematical convenience we treat, as a first step, the $\vec{\ell}$-independent condition. Furthermore, we restrict ourselves to symmetric fission with spherical fragments. The absolute values of the oscillator frequencies ω are obtained via the condition of surface volume conservation which imposed that the equipotential surface which coincides with the nuclear surface was kept constant during the deformation. The single particle energies of the four-center shell model for some light nuclei (^{16}O, ^{32}S, ^{48}Kr, ^{64}Ge) and one heavy nucleus are obtained numerically. It is convenient to calculate the single particle energies for one light nucleus analytically. By adding the single particle energies up to the fermi-level, a rough estimation of the binding energy depending on the deformation parameter (a) can be obtained. This will be shown for ^{16}O, ^{32}S, ^{48}Kr, ^{64}Ge and ^{208}Pb. The remaining problem is the generalization of this model to allow the break up into four different fragments and an appropriate liquid drop calculation to the general shapes [19].

The present thesis contains mainly four chapters. The first chapter represents a review on the previous multi-center model. The second chapter describes in some detail the methods of the two-, three- and four-center oscillator potential arranged in a line (chain configuration). The third chapter deals with the four-center oscillator arranged in a plane and the method of calculation of the single particle energies both numerically and analytically. The fourth chapter contains the results of calculation and discussion. Details of some calculation are given in the Appendix.

CHAPTER (1)

REVIEW ON MULTI-CENTER MODELS

1.1 Double-Center Oscillator Model

The Nilsson model [4] becomes invalid for large deformation in the sense that it does not contain the proper asymptotic behaviour required in fission process: instead of allowing the formation of two separate fragments, the Nilsson model approaches asymptotically only long, stretched, cigar like shapes ("needle ellipsoids") for the equipotential surface as shown in Fig. (1). The resulting increase in the single particle zero-point energy gives rise to the general trend of increasing the single particle energies as the deformation increase. In particular, the collective potential energy surface, which is essentially a sum of single particle energies, will approach infinity for large deformation and, therefore, it will not give a fission barrier. This question leads to the development of an asymptotically correct shell model which is based on the double-center model [7-13]. Since the surface of the two center shapes model does not go to infinity for large deformation in contradiction to the case of the one-center model, Holzer, Mosel and Greiner have shown in their work [7] the importance of the two center single particle potential as a starting point for all microscopical fission calculations. They have chosen in particular oscillator potentials, since

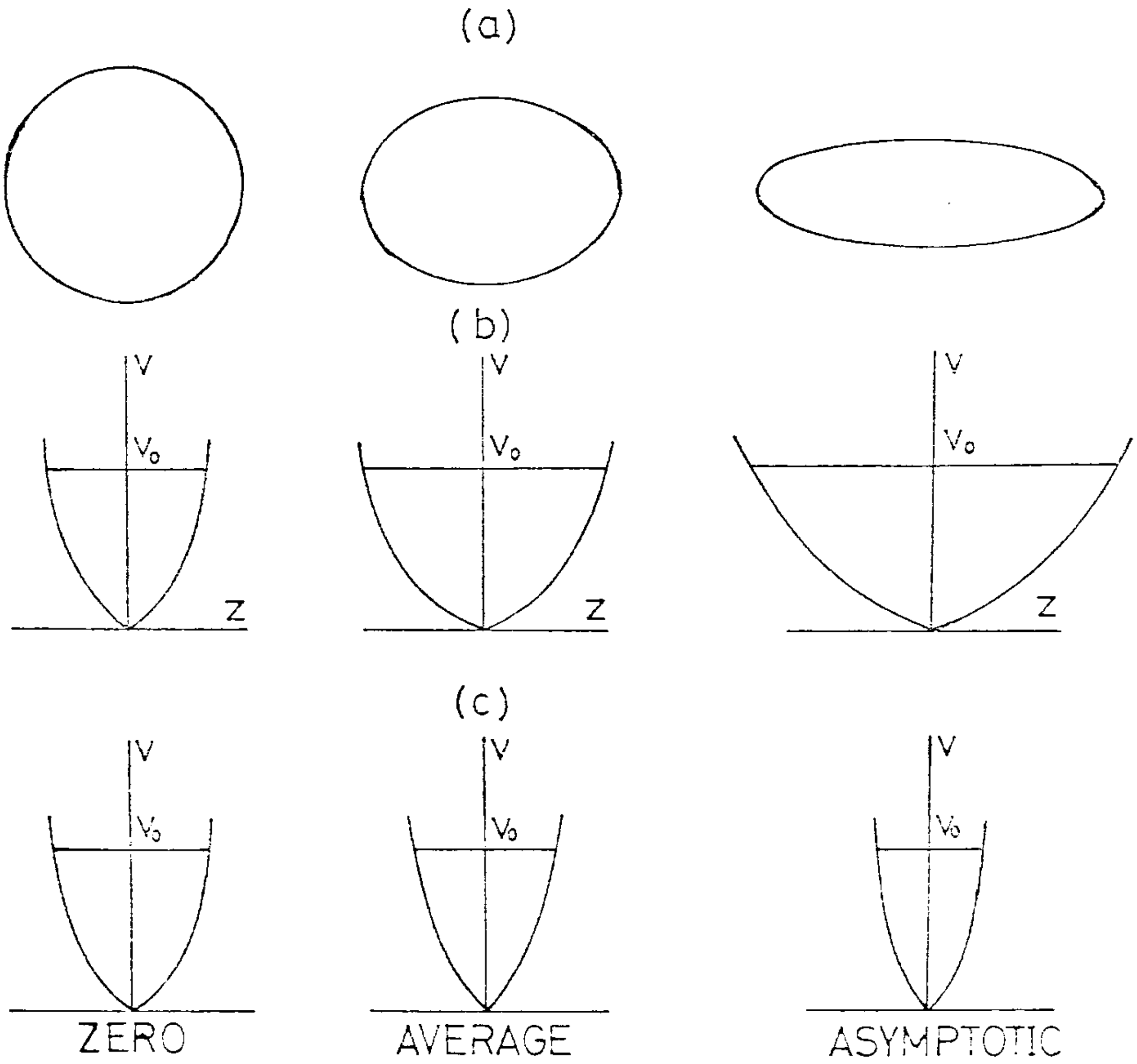

Fig.1. Nilsson model for three deformation (Zero, Average, Asymptotic). V_o is the potential at the surface of the nucleus which is equipotential.

(a) Surface of the nucleus
(b) Potential in z-direction
(c) Potential in ρ-direction

they can be handled mathematically and are also underlying the conventional shell model. They started with a Hamiltonian which is a direct generalization of the Nilsson model to two center potential and has the following form in cylindrical coordinates:

$$H = T + V(\rho, z) + V(\vec{\ell}_1, \vec{\ell}_2) \tag{1.1}$$

with

$$V(\rho, z) + V(\vec{\ell}_1, \vec{\ell}_2) = \frac{m}{2} \begin{cases} \omega_{1\rho}^2 \rho^2 + \omega_{1z}^2 (z - z_1)^2 + C\vec{\ell}_1 \cdot \vec{s} + D\vec{\ell}_1^2, & z > 0 \\ \\ \omega_{2\rho}^2 \rho^2 + \omega_{2z}^2 (z + z_2)^2 + C\vec{\ell}_2 \cdot \vec{s} + D\vec{\ell}_2^2, & z < 0 \end{cases}$$

where $\vec{\ell}_1$ and $\vec{\ell}_2$ are the angular momentum operator with respect to the two centers z_1 and z_2 respectively. It is noticed that an appropriate choice of the frequencies and the parameter C and D yields the proper transition from the spherical one-center model to the spherical double-center model.

Figure (2) shows the geometrical shapes which described the double-center in different cases. They have treated, as a first step, only the $\vec{\ell}$-independent terms of the Hamiltonian and restricted themselves to the spherical symmetric case i.e. $\omega_{1\rho} = \omega_{2\rho} = \omega_{1z} = \omega_{2z}$. The geometrical shapes corresponding to these cases for various stages of separation of the two centers are shown in Fig. (3). From the figure and the Hamiltonian