THE STABILITY OF PLANE FRAMEWORKS

(Application to Rigidly Jointed Plane Trusses)

A THESIS

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By

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LIST OF SYMBOLS

The following symbols are used in this thesis. Other symbols not listed below are defined where they are used :

A = cross-sectional area ;

c = nondimensional carry-over factor for fixed end member ;

$\bar{c}$ = modified carry-over factor for members having gussets of infinite rigidity ;

E = Young's modulus of elasticity ;

F = equilibrium forces ;

f = out of balance forces ;

g_1 & g_2 = length of gusset plates at end 1 and end 2 of member ;

I = moment of inertia ;

k = EI/L = bending stiffness , and in the presence of rigid gusset plates $k = EI/l$;

L = unstrained length ;

L_c = strained length ;

L_a = arc length ;

l = $(L - g_1 - g_2)$;

L_b = buckling length ;

L_{bg} = buckling length taking into account effect of rigid gussets ;

M = bending moment ;

P_o = external panel load ;

P = axial load ;

P_E = $\pi^2 EI/L^2$ = Euler's load , and in the presence of rigid gusset plates P_E = $\pi^2 EI/l^2$;

S = shearing force ;

s = nondimensional stiffness factor for fixed end member ;

$\bar{s}$ = modified stiffness factor for members having gussets of infinite rigidity ;

Truss = rigidly - jointed plane truss;

X = projected length of member along the x-axis ;

x = deflection along the x-axis ;

Y = projected length of member along the y-axis ;

y = deflection along the y-axis ;

α = initial angle of inclination of a member with the x-axis ;

$\bar{\alpha}$ = modified value of angle α ;

Δ = an estimated value for η ;

η = final displacements ;

ϵ = axial displacement ;

θ = angle of rotation with respect to initial direction of member ;

ρ = P/P_E ;

ϕ = angle of sway with respect to initial direction of member.

INTRODUCTION

General :

Existing methods of design used for steel structures and based on elastic theory are empirical methods of unknown accuracy. The natural growth of these methods has led to comparing the behaviour of struts in structures with that of isolated struts. Thus, the buckling length of any member was assumed a constant fraction of its theoritical length. These methods have not led to anything more than empirical methods which may be over-generous. Therefore, it is necessary to find the elastic critical loads for a complete structure. This is not difficult when calculations are carried out by means of the electronic computers.

The thesis presented is carried out with the intent to two main objects. The first is to determine the elastic critical loading pattern, as well as the buckling lengths of a sets of rigidly-jointed plane trusses for various stiffnesses of upper chord, lower chord and web members to be considered as a design data for civil Engineers. The second is to study the effect of all the secondary effects on the elastic critical loads of rigidly-jointed trusses.

Pin-jointed, statically determinate trusses present no problem, since axial loads are directly proportional to the applied loads. Each compression member acts as a pin ended strut for which the critical buckling load is equal to its Euler load ($P_{cr} = P_E = \pi^2 \; EI/L^2$). Any combination of loading, which will cause the stress in any of the compression members to reach its critical value, will cause failure of the whole truss.

Rigid-jointed statically determinate trusses (i.e. trusses that are statically determinate in their primary stresses) are the most fruitful for discussion because of their practical importance. The compression members in such trusses will not act independently because of the secondary moments involved and the buckling loads of these members are not reached untill the combination of loading on the truss causes instability of the whole truss. To a close approximation, the axial loads in such trusses are proportional to the applied loads.

Rigid-jointed statically indeterminate trusses differ from statically determinate trusses as the axial forces are not proportional to the applied loads. This due to the inclusion of the secondary effects.

Four programmes have been constructed by the author. Detailed write-up of these programmes requires a lot of space , therefore, it has been decided to describe the first programme in full details and to describe the main lines of the other programmes with the aid of flow diagrams to show how the solution can be carried out.

Three methods of analysis are performed. The first is a simplified method which neglects the joint translation and the axial forces in the truss members are calculated on the assumption of pin-jointed connections. This method is based on the investigation of distorted configuration of the truss related to chosen disturbances. The second is an accurate analysis which studies the effect of joint translation and the change in the axial forces due to the shear in the members caused by the secondary moments. The third is a more accurate analysis which includes the following effects :

1- Effect of joint translation ;
2- Effect of the change in the axial forces due to the shears in members ;
3- Effect of the change in geometry of the deflected truss, i.e. the change in the angles of inclination and lengths ;
4- Effect of the difference between the arc length and the strained length of members (bowing effect).

The second and the third methods of analysis are based on the investigation of the stiffness matrix for the whole truss related to the actual applied loads.

General Assumption :

The study is based on the following basic assumptions :

1- Buckling outside the plane of the truss is prevented.

2- The truss is perfectly elastic.

3- The deflections of the members are small.

4- The members are initially perfectly straight and without end eccentricities.

5- The centroid and the shear centre of any cross-section both occur in the plane of the truss. This condition satisfied for plane trusses with loads acting in the plane of the truss only, the members all having axes of symmetry within that plane. (The stability functions s & c may only be applied to such a member if it is so constrained continuously along its length that twisting about the longitudinal axis is everywhere prevented).

The first assumption means that any member is undergoing flexure about one principal axis only, and that bending about the other principal axis (i.e. out of the plane of the truss) does not occur. The second assumption means that the stress-strain relationship is assumed to be indefinitely elastic. The third assumption involves the governing differential equations which express the relationship between bending moment and deflection. Let us examine the following expressions :

$$\frac{M}{E\,I} = -\frac{d^2y}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(a)}$$

and $$\frac{M}{E\,I} = -\frac{d^2y}{dx^2} \Big/ \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \quad \ldots\ldots \text{(b)}$$

Equation (b) is the more exact relationship. But equation (a) is a good approximation if the deflection is small. The third assumption implies that the equation (a), not equation (b) is used for the analysis. The fourth and fifth assumptions also limits the scope of the present work.

Scope :

This thesis consists of five chapters :

Chapter I presents a review of the previous work. Most of the work available up till now which deals with the critical loads of complete trusses is given.

Chapter II deals mainly with the determination of the buckling loads and buckling lengths of a set of statically determinate rigidly-jointed plane trusses according to the simplified method of analysis which is a very successful method for such trusses. The effect of infinitely rigid gusset plates is also included.

In chapter III the additional effects of joint translation and shearing forces on the buckling loads of trusses are studied for the cases of statically determinate