فى الشق الأول تم حل النظام فى حالة الموجات الطويلة ( الأعداد الموجية الصغيرة ) ، وطبقت الشروط الحدية عند سطح الانفصال ، فوجد أن استقرار الاتزان للنظام يعتمد على معاملات السرعات . والمناقشة البيانية لاستقرار الاتزان بينت أن المجال الكهربى الثابت ، العمودى على سطح الانفصال له تأثير غير استقرارى للاتزان للاضطرابات الناشئة عند سطح الانفصال ، وأن تأثير المجال الكهربى يقل بدرجة كبيرة بزيادة سمك طبقة المائع السفلى . ودرست الحالات الخاصة لسريان بواسل وسريان كويت لطبقتى مائعين ، متراكس ، مختلفى اللزوجة ، ومحصورين بين مستويين أفقيين . ووجد أن كلا من سريان بواسل وسريان كويت يحدث عدم استقرار الأثر ان للاضطرابات الناشئة عند سطح الانفصال ، مهما كانت قيمة عدد رينولدز.

أما فى الشق الثانى فتم حل النظام فى حالة قيم عدد رينولدز الصغيرة، وطبقت الشروط الحدية المناسبة عند سطح الانفصال . وهنا تمت دراسة استقرار الاتزان للنظام فى حالتين: حالة الوسط الذى يكون فيها المائع السفلى أثقل من المائع العلوى . ووجد أن المجال الكهربى الثابت ، العمودى على سطح الانفصال له تأثير غير استقرارى للنظام . كما وجد أن القيم الحرجة للمجهد الكهربى تكون صغيرة عند قيم الكثافة الموحدة ، وأنه عند القيم الحرجة للمجهد الموحد يحدث عدم استقرار الاتزان للنظام ، حتى فى حالة غياب المجال الكهربى . وأن مستوى استقرار الاتزان يكثر بزيادة النسبة بين عمقى الطبقتين السفلى والعليا . أما الحالة الثانية وهى التى يكون فيها المائعان متساويين فى الكثافة ، فإنه بالرغم من أن المجال الكهربى يسلك طريقة مشابهة للحالة الأولى فإن عدم استقرار الاتزان للنظام يبدأ أسرع ، لأن القيم الحرجة للأعداد الموجة تكون أصغر فى هذه الحالة .

تمت بحمد الله

# ELECTROHYDRODYNAMIC STABILITY OF TWO SUPERPOSED FLUIDS



## A THESIS

*Submitted in Partial Fulfilment of the Requirements of the Award of the Master of Science Degree*

*By*

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*Faculty of Science*
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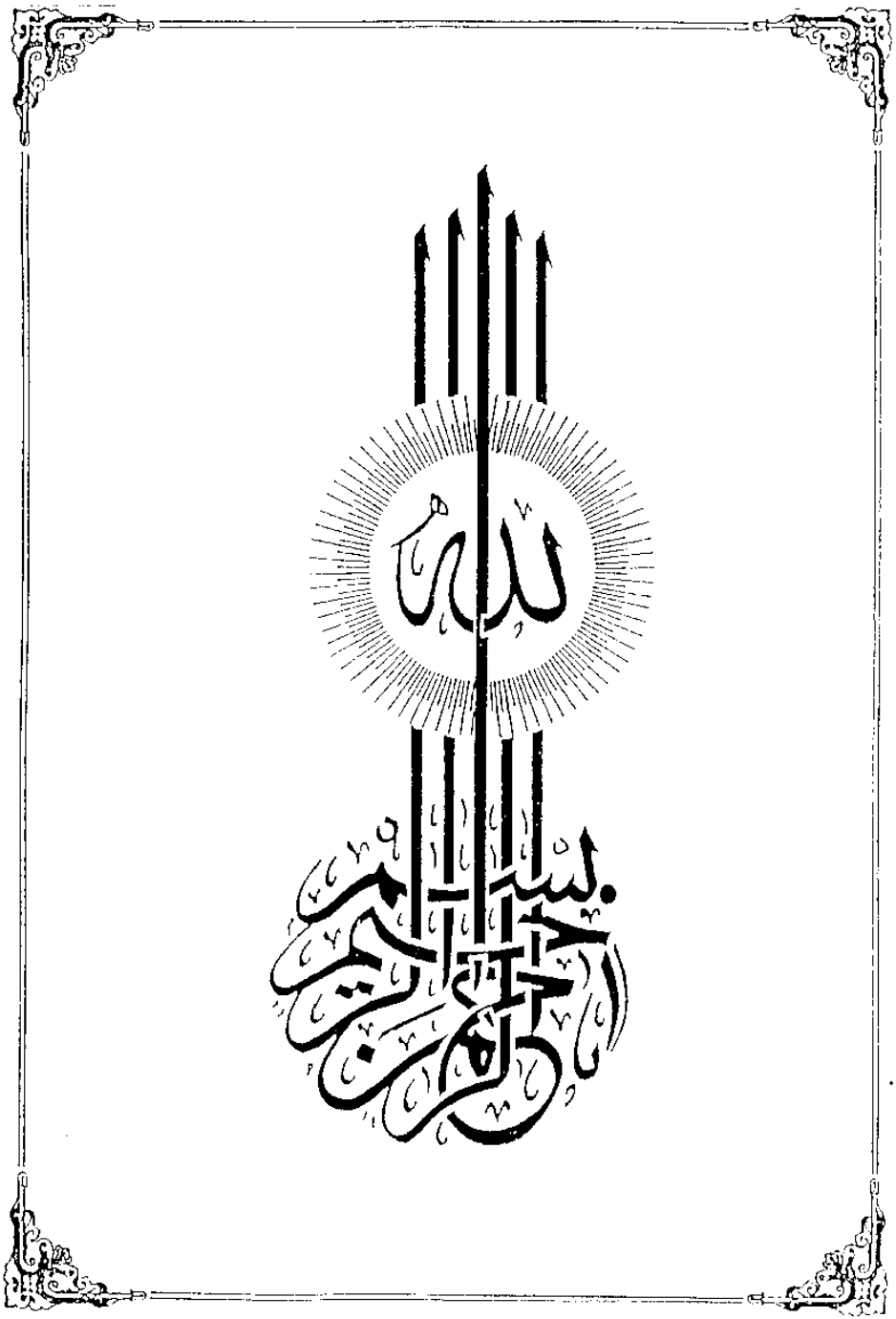
1984



بسم الله الرحمن الرحيم

قالوا سبحانك لا علم لنا إلا ما علمتنا

إنك أنت العليم الحكيم

( صدق الله العظيم )

ACKNOWLEDGEMENTS

I would like to express my deep appreciation to my supervisor Prof. ABOU EL - MAGD A. MOHAMED, professor of Applied Mathematics, Faculty of Education, Ain Shams University, for suggesting the problems involved in this work and for his helpful guidance, valuable suggestions and discussions throughout his supervision of this work.

I am very grateful to my co - supervisor Dr. EL - SAYED F. EL - SHEHAWEY, lecturer of Applied Mathematics, Faculty of Education, Ain Shams University, for his stimulating advice and continuous encouragement throughout his supervision of this work.

I am deeply indebted to Prof. E.M. EL - GAZZY , Head of the Department of Mathematics, Faculty of Education, Ain Shams University, for his interest in this work. His encouragement is greatly acknowledged

I am grateful to Prof. D.H. MICHAEL, University College, London , and Prof. D.G. CRIGHTON, University of Leeds, England, for their discussions during their visit to the Department of Mathematics, Faculty of Education, Ain Shams University.

My thanks are also due to the Head and the Staff of Applied Mathematics Department, Faculty of Science, Ain Shams University for their kind cooperation and facilities offered throughout this investigation.

# N O T E

The present thesis is submitted to Ain Shams University in partial fulfilment of the requirements of the Degree of Master of Science in Applied Mathematics.

Beside the research work materialized in this thesis, the candidate has attended six postgraduate courses within a year (1980 - 1981) including the following topics :

(1) Theory of Stability.

(2) Fluid Dynamics.

(3) Electromagnetic theory and Magnetohydrodynamics.

(4) Theory of Elasticity.

(5) Classical Mechanics.

(6) Numerical Analysis.

Th applicant MOHAMED FAHMY EL - SAYED AHMED , has successfully passed the final examination of these courses.

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TO

THE MEMORY OF

MY FATHER ( 1927 - 1979 )

CONTENTS

# SUMMARY

# SUMMARY

The thesis is mainly concerned with the electro - hydrodynamic stability of an interface separating two fluids. The upper fluid is conducting incompressible fluid and the lower one is dielectric incompressible fluid, which is subjected to a constant electric field and directed perpendicularly to the interface. The fluids are stressed by gravity force and there exist surface charges on the interface.

The following problems are investigated :

(1) Electrohydrodynamic Kelvin - Helmholtz Instability For a Velocity Stratified Inivscid Fluid With Variable Density.

(2) Electrohydrodynamic Stability of Two Superposed Viscous Fluids With Velocity Stratification.

In chapter one we introduce the main aspects, the previous works of electrohydrodynamics and its various applications. We explain the basic concepts of electro - hydrodynamic stability and the fundamental aspects of the topic. We introduce the equations governing the motion and the boundary conditions which are satisfied by the solutions. We also discuss the basic concepts of stability and the

techniques followed in this respect.

In chapter two we introduce the previous works in the subject of the Hydrodynamic stability of two superposed (viscous or stratified inviscid) fluids. We also introduce the Orr - Sommerfeld equation and the Rayleigh equation which governs the stability of viscous and inviscid flows respectively. The Squire's theorem which indicates the relation between the behaviour of two - dimensional disturbances and that of three - dimensional ones for incompressible fluids is presented. We explain the previous works in solving the Orr-Sommerfeld equation and the inviscid limit which indicates the relation berween the solutions of the Orr - Sommerfeld equation in the limit of vanishing viscosity and the solutions of the inviscid equation.

Chapter three deals with the electrohydrodynamic stability of a streaming conducting incompressible stratified inviscid fluid which extends to infinity topping a dielectric incompressible inviscid fluid layer with finite depth which is at rest. The lower fluid is bounded from below by a rigid conducting plane and is subjected to a constant electric field which is directed

perpendicular to the interface. The fluids are stressed by gravity force and there exist surface charges on the interface. We solve the equations of motion governing a small disturbance to which the system is subjected (these are two differential equations, the first of which is of the Whittaker's type, and the second equation is a second order ordinary differential equation). Application of the boundary conditions leads to the dispersion equation. We discuss the stability of the system theoretically and numerically. We also discuss the special case of the Rayleigh - Taylor instability and we obtain the necessary and sufficient conditions for stability.

It is found that the normal electric field has a destabilising effect while the increase of the thickness of the layer has a stabilising influence.

Chapter four is concerned with the electrohydro - dynamic stability of a streaming conducting viscous incompressible fluid layer topping a streaming dielectric viscous incompressible fluid layer between two horizontal plates. The lower fluid is bounded from below by a rigid conducting plane and is subjected to a constant electric field.The fluids are stressed by gravity force and there