

Modeling and Control of an Inverted Pendulum Based on Evolutionary Computational Techniques

By

Mohamed El Sayed Mousa Afefi

A Thesis Submitted to the
Faculty of Engineering at Cairo University
In Partial Fulfillment of the
Requirements for the Degree of
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in
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Title of Thesis:

Modeling and Control of an Inverted Pendulum Based on Evolutionary Computational Techniques.

Key Words:

PI, PID, FOPID, Linear Quadratic Regulator (LQR), Inverted Pendulum (IP), Particle Swarm Optimization (PSO) technique.

Summary:

Inverted pendulum is one of the most excited problems in dynamics and control theory. It's Single Input Multi Output (SIMO) problem. PI, PID and FOPID controllers with feed forward gain based on Reduced Linear Quadratic Regulator (RLQR) are applied to the IP system. Tuning the Controllers' gains is achieved by using Particle Swarm Optimization (PSO) technique. The objective function is designed to give minimum settling time, rise time, undershoot and over shoot for both the cart position and the pendulum angle.

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When they say give up!

When there isn't anything in the life encourage you for achieving your dreams. Each day ended with the same criteria. Due to God and some persons whose presented a lot of help for me, the life changes. The journey towards reaching your aims in research fields is highly tormenting and difficult. The researcher is mainly depending on the hope in his research. It is the element that completed the research work. Throughout my life, I inspire my hope from a beacon of light which always give me hope when i was disappointed.

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