AIN SHAMS UNIVERSITY
UNIVERSITY COLLEGE OF WOMEN FOR ARTS, SCIENCE AND EDUCATION
MATHEMATICS DEPARTMENT

" THEORETICAL INVESTIGATIONS ON FUSION REACTIONS "

A Thesis Submitted For The Degree of
Doctor Of Philosophy (Ph.D.)
Applied Mathematics

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صدق الله العظيم

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INTRODUCTION

In Chapter I, the two main methods of studying the two body scattering are outlined. First the successive Born approximation, which is obtained by using the Green function procedure, expressed in the k_0 complex plane. And that following the neat method of Lippman-Schwinger[1], written in symbolic form. The second method is the famous partial wave expansion method, where the solution of the Schrödinger equation is expressed as a sum of partial waves, each being the product of a function of angles, and a function of **r.** This method of partial waves can also be extended to using other coordinates, for example elliptic coordinates $\mathbf{u},\mathbf{v},\chi$ instead of spherical polar coordinates $\mathbf{r}, \theta, \chi$. An exact expression is obtained when the mutual potential is assumed to vanish for distances greater than certain distance, depending on certain phase angles.

At the end of this chapter, an important example is investigated dealing with the scattering of a charged particle by an extended spherical nucleus of radius a. It is known that for the investigation of the interior structure of nuclei, high energy electrons (from the high energy accelerators) have been used to obtain experimental results for the differential scattering cross section, from which, the interior structure of the nucleus can be obtained. A simple model for the interior structure of the nucleus has been assumed in which the nucleus is considered as a uniformly charged drop of finite radius. The potential inside is thus considered to be the Coulomb potential inside a uniformly charged sphere plus a negative constant potential due to the nuclear forces inside a nucleus.

The solution of the Schrödinger equation for such a potential inside the nucleus happens to be given exactly in terms of hypergeometric functions multiplied by an exponential. Using the condition that the solution must be finite at the centre of the nucleus leads to discarding one of the solutions, Thus we have only one solution depending on one arbitrary constant inside the nucleus. For the solution outside the nucleus, where the potential is just the Coulomb potential $\frac{Z}{r}$, we as usual have two solutions. One solution being the usual one depending on a hypergeometric function, and the second is quite elabortate[2], and have been given incorrectly in the literature. Applying the condition at infinity, these two solution have been given depending only on one arbitrary constant. The two arbitrary constants for the solutions inside and outside are then obtained using the boundary conditions on the surface of the drop.

To check the results, another method have been applied, on expressing the second solution outside the nucleus as an integral over an integrand depending on the first solution. Numerical application have been given for the nucleus $^{40}_{18}Ar$, when the scattered charged particle is $^{4}_{2}He$.

In chapter II, the problem of the scattering of neutrons by a nucleus is investigated, and that on using the partial wave expansion method. First by considering the nucleus as spherical, and secondly by considering the nucleus as a uniformly charged drop bound by a surface in the form of an ellipsoid of revolution[3].

The same problem has been investigated, using the W.K.B. approximation. An example is computed in the case $\ell = o$, $\gamma k = 2$, outside and $\gamma K = 3$ inside.

In chapter III, the case of a complex scatterer is investigated. First, considering the scatterer as a quantum system defined by internal coordinates ξ and an internal hamiltonian $H(\xi)$. The problem is fully investigated on using the Green function method, already discussed in chapter I. The open and closed channels for the scattering are clearly defined, and the scattered wave is obtained as an integral equation to be solved successively as in the successive Born approximation method. Next, considering the scatterer as a continuous distribution, we use the drop model of the nucleus. First we consider the nucleus, approximated by a tri-axial uniformly charged ellipsoid, and consider the variation of the surface and kinetic energies under the action of the internal Coulomb and surface interactions alone.

An example is given, considering the scattering of a charged particle by a uniformly charged drop in the form of an ellipsoid of revolution.
The Schrödinger equation is solved using elliptic coordinates u, v, χ , as a sum of partial waves. In the limiting case when the drop is nearly spherical, we can consider the scatterer as a quadrupole. In this case the solution of

The Schrödinger equation is given. A classical solution, using the classical hamilton equations of motion is also shown.

Finally, in chapter IV, the use of scattering formula in the study of nuclear reactions are investigated. The reaction probability and the reaction power rate are clearly defined. The equations giving the time rate of the concentrations of a reacting system having different reaction channels is given.

An example of an initial mixture of deutrons or of deutrons and tritium is illuatrated by considering the possible reaction channels.

The solution of the concentration rate of different components, can however be only solved numerically as the equations are non-linear in the concentrations. Taking numerical values for the different reactions rates in the different reaction channels, a computation program has been carried out to find the different concentrations at any subsequent time.

The average of $<\sigma v>$ for different reaction channels his been taken from previous computations(16),(18). Using an approximate expression for the reaction scattering cross section as given by Equation (4.10). In chapter V, the numerical computation for the different concentration has been extended up to t = l second for $T = 10^8$ K for an initial pure deutron number concentration of 10^{18} cm^{-3}. Also the power of reaction (as given by Eq. (4.29) is computed for the same time intervals.

In the same chapter, a rigorous treatment of the rate of energy dissipation through Bremstrahlung (scattering of electrons in the external field of nuclei with the subsequent emission of photons) is carried out using relativistic statistical mechanics of the electron gas in the plasma.

This energy rate has however been given approximately as proportional to the square root of the absolute temperature T [18].

The rate of Bremstrahlung energy dissipation has been computed numerically and found that it is always bigger than the power rate of the nuclear reactions as shown in table (5.3). Also the numerical computation is repeated for the same initial number concentration of pure deutron for two different temperature $T = 1.16 \times 10^8$ K and $T = 11.60 \times 10^8$K where the average of $<\sigma v>$ have been taken from ref(18). In this latter case, the reaction power rate becomes bigger than the radiation dissipation rate for $T = 11.60 \times 10^8$K. It may be noted that the number concentration of tritium is almost negligable as shown from table (5.2) and figure (2), when up to t = 1 second. The resulting bigger neutron concentration can be utilized by surrounding the reaction enclosure by ${}^6_3Li$ blanket [21] $({}^1_0n + {}^6_3Li \rightarrow {}^4_2He + {}^3_1H)$. (Notice that ${}^6_3Li$ forms only 7.5% of natural lithium).

The relativistic average kinetic energy of the electron gas in the plasma has been computed and found that it deviated from the classical value 3/2 kT to bigger values as T increases. This means that at the same temperature the average kinetic energy of an electron is bigger than the average kinetic energy of any nucleus. Also the spectrum of the Bremstrahlung radiation (as computed to the total radiation rate) is compted.Using a rigorous relativistic expression and found that it always decreases with an increase of $\omega = h\nu/m_e c^2$ and almost vanishes for $\omega = 0.2$. Although the Bremstrahlung radiation is treated rigorously the reactions power rate is computed using approximate expressions (16), (18) for the reaction scattering cross section . There are however many factors to modify the reaction cross section between the nuclei ${}^2_1H, {}^3_2He$ (5), (6), (17) ....which are not considered in our study in this thesis.

This thesis ends with eight mathematical appendices to clarify the different mathematical operations carried out in the thesis.

CHAPTER I

TWO BODY SCATTERING