A THESIS
ENTITLED

STUDIES ON CONTROLLED SPATIAL-FREQUENCY MULTIPLE-BEAM INTERFERENCE FRINGES

PRESENTED

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BY

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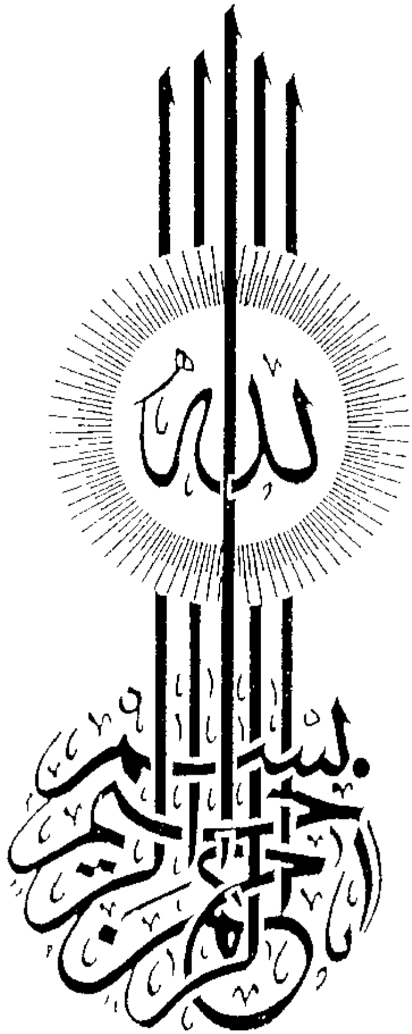
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ABSTRACT

Two-beam and multiple-beam interference fringes of controlled spatial frequency and known characteristics provide tools for a variety of applications. Methods of fringe formation on planes of localization in space and their characteristics have been presented and performed, with emphasis on spatial frequency control. Such studies dealt with intensity distribution of multiple-beam fringes formed on Feussner surface of zero order of localization for a silvered air wedge and a wedge enclosing a dispersive medium, taking into account the phase change, the light suffers, at reflection air/metal, medium/metal. The present studies dealt also with fringe systems formed on higher order planes of localization far from the silvered wedge interferometer and those formed on fractional orders of localization on both sides of Feussner surface of zero order. Fine structure in fringe systems on integral and fractional order planes of localization for limited number of interfering beams is reported.

Masking all the images of the illuminating source but two, leads to the formation of fringe systems of two-beam intensity distribution of controlled spatial frequency over a wide range of frequencies and of known visibility. This provides a sinusoidal source suitable for measuring the optical transfer function for the evaluation of lens performance.

CONTENTS

CHAPTER III

TWO BEAM FRINGE SYSTEMS OF CONTROLLED SPATIAL FREQUENCY FOR EVALUATION OF LENS PERFORMANCE

CHAPTER I

FORMATION OF MULTIPLE BEAM INTERFERENCE FRINGES BY A SILVERED WEDGE ON HIGH ORDER PLANES OF LOCALIZATION AND THEIR SPATIAL FREQUENCY

I.1- Introduction :

Multiple-beam interference takes place between two optical-flats coated with highly reflecting thin films. The resultant interference fringes are affected by the reflection coefficient of the coated surfaces, the geometrical condition of the interferometer, and the nature of the light used [1].

A wedge interferometer is formed when the two optical flats forming the interferometer are inclined to each other making a small angle. Two-beam interference takes place when the two surfaces are uncoated. This interferometer was utilized by Fizeau [2] in 1862. Tolansky [3] used highly reflecting coated surfaces. In this case multiple-beam interference takes place in transmission [4] and at reflection [5]. With monochromatic point source, these fringes in transmission are sharp bright straight lines on dark background. The separation, between these fringes depends on the wedge angle. The refractive index of the medium enclosed between the two coated components of the interferometer, and the wavelength of the light source used.

Multiple-beam interference fringes are formed on a surface close to the interferometer surface called the Feussner surface [6] which is the zero order plane of localization. Interference fringes can also be formed at certain distances away from the interferometer, i.e., on high planes of localization [7]. These planes are equally spaced [8].

I.2- Methods of Calculating the Phase Difference Between the Direct and n^{th} Beam Which Has Suffered 2n Reflections :-

Tolansky [9] in 1946 carried out the necessary conditions required to produce multiple-beam Fizeau fringes using a wedge interferometer. He pointed out that the successively multiple reflected beams are not behind each other in phase, they are not in exact arithmatic series as in the case of two parallel plates. There is an increasing phase lag which depends on the number of reflections the beam suffers, the thickness of the wedge and the wedge angle. When this phase lag exceeds π the fringe sharpness and intensity will drop.

The phase condition can be derived by three different methods which are : 1- wavefront method, 2- Ray tracing method, 3- Stretching method.

These method are proved here in case of normal incidence considering the cases of dispersive and nondispersive media. In the present work, the phase changes that light suffers at air/metal and dispersive medium/metal have been taken into consideration. This is firstly considered in this work.

I.2.1. Plane Wavefront Method :

Brossel [7] in 1947 has simplified the calculation for the phase condition and generalized it to include regions other than those on the Feussner surface close to the wedge surface. The calculation depends on consideration of plane wavefronts.

I.2.1.a- The Case of a Silvered Air Wedge :

Consider two highly reflecting surfaces M_1 and M_2 with a small angle between them (Fig. 1.1.a)Let a monochromatic planewave be incident normally on the air wedge. The incident plane wavefront suffers multiple reflections inside the interferometer which gives a family of plane wavefronts D_o, D_1,..., D_n. The resulting wavefronts have intensities decreasing geometrically and the angle betrween successive members being 2ε. Considering cartesian coordinates (x,y), the origin (o,o) is the apex of the wedge and the y-axis coincides with M_1. Let p be some field point having coordinates (x_o, y_o).

The path difference between the directly transmitted beam arriving at p and the n^{th} beam which has suffered 2n reflections is :

$$\delta_{2n} = pW_n - pW_o + 2n\left(\frac{\lambda\beta}{2\pi}\right) \qquad (1.1)$$

W_n, W_o are the feet of the perpendiculars from p to D_o and to D_n, respectively. β is the phase change arising from a single reflection at either surfaces of the wedge. The phase change upon transmission is omitted, since it is equal for the two considered beams.

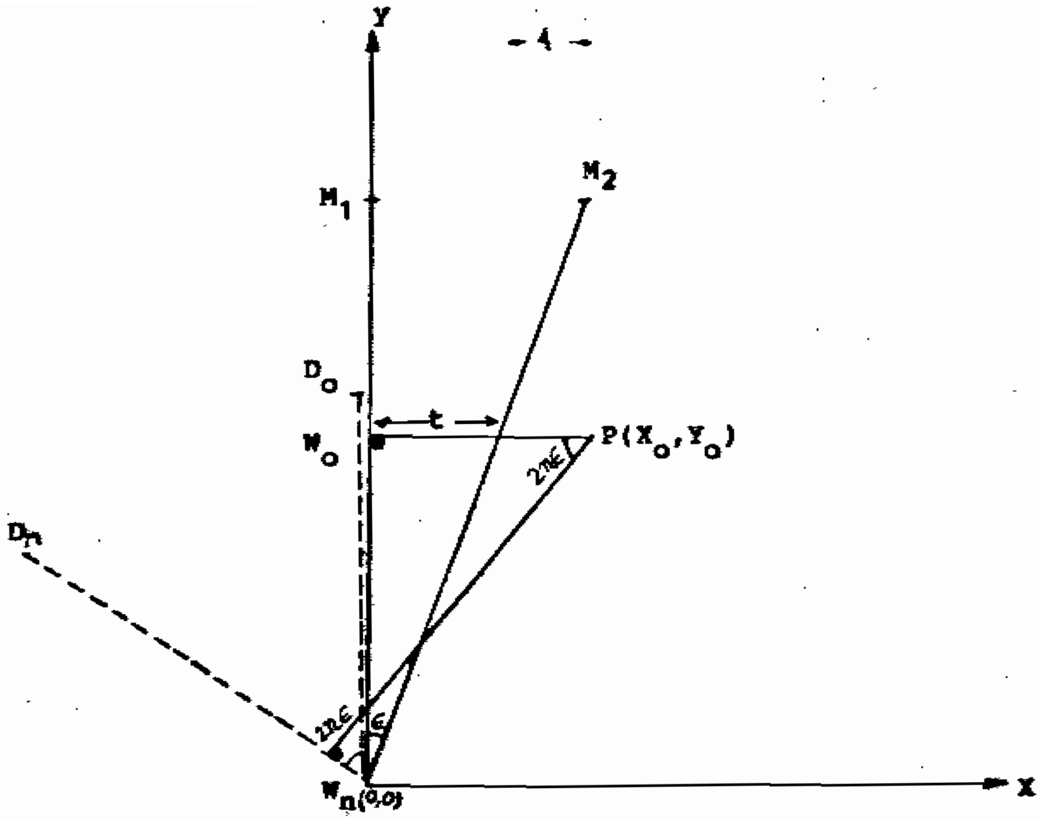

Fig. 1.1.a. Wavefronts after multiple reflections in an air Wedge

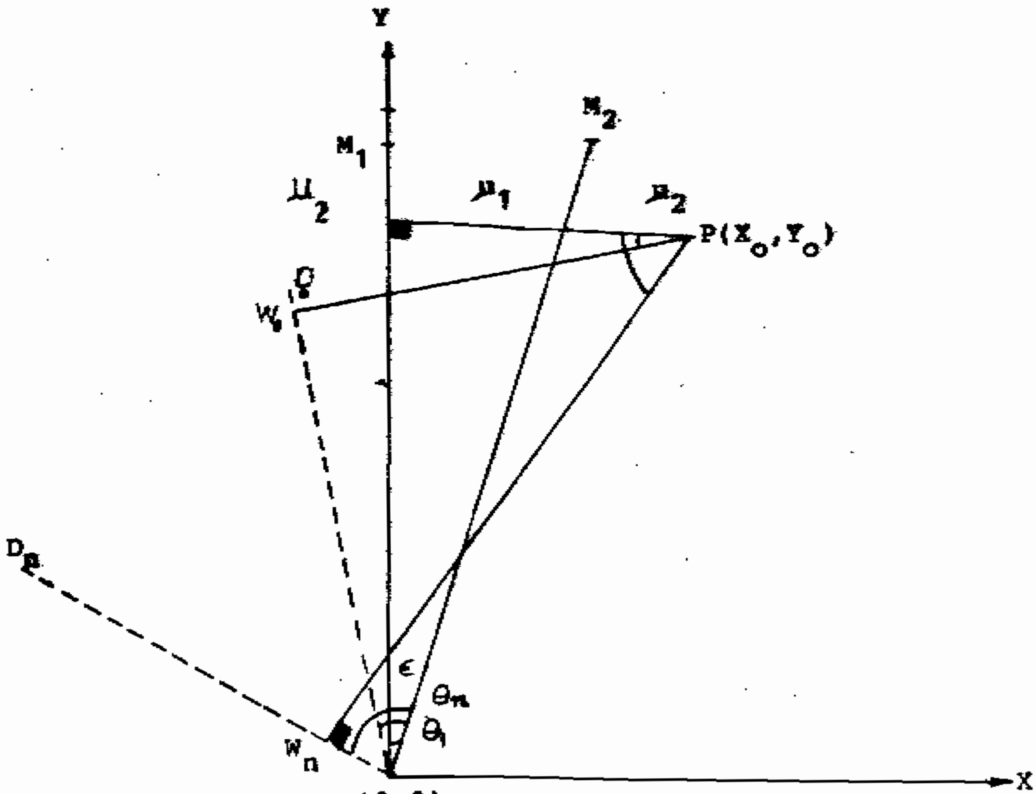

Fig. 1.1.b. Wavefronts after multiple-reflection in a silvered wedge enclosing a dispersive medium.

Since

$$PW_n - PW_o = X_o \cos 2n\varepsilon + y_o \sin 2n\varepsilon - X_o$$

$$= X_o(\cos 2n\varepsilon - 1) + y_o \sin 2n\varepsilon$$

Then,

$$\delta_{2n} = X_o(\cos 2n\varepsilon - 1) + y_o \sin 2n\varepsilon + 2n(\frac{\lambda\beta}{2\pi}) \quad (1.2)$$

We may expand equation (1.2) in powers of ε and retain terms only up to the second power. We have,

$$\delta_{2n} = 2nt\ [1 - \frac{2n^2+1}{3}\ \varepsilon^2] - 2n^2\varepsilon^2 X_o + 2n\ (\frac{\lambda\beta}{2\pi}) \quad (1.3)$$

where $t = y_o \tan\varepsilon$ and t is the wedge thickness at the point of incidence. If we neglect the effect of phase change upon reflection, the path difference near the interferometer surface is $2nt[1 - \frac{2n^2+1}{3}\ \varepsilon^2]$. This value differs from that of a Fabry-Perot interferometer by an increasing retardation term $\frac{2}{3}\,nt(2n^2+1)\varepsilon^2$. The effect of this term is greatly reduced if ε is small enough and the reflectivity of mirrors is not high.

I.2.1.b- <u>The case of a silvered wedge enclosing a dispersive medium</u> :

Born and Wolf [10] derived the phase conditions for a wedge enclosing a dispersive medium. This derivation considered only the Feussner zero order plane of localization. In the present work, we shall consider the zero order and the higher order planes of localization. Consider a wedge enclosing a dispersive medium of refractive index μ_1, and the refractive index of the surrounding medium being μ_2 (Fig. 1.1.b). The transmitted wavefronts D_o, $D_1, \ldots, D_n$ will emerge at angles $\theta_1, \ldots, \theta_n$. From the Snell's law we get :

$$\mu_1 \sin\varepsilon = \mu_2 \sin\theta_1, \quad \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}\sin^2\varepsilon} = \cos\theta_1$$

and

$$\mu_1 \sin(2n+1)\varepsilon = \mu_2 \sin\theta_n, \quad \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}\sin^2(2n+1)\varepsilon} = \cos\theta_n$$

The path difference between the directly transmitted wavefronts arriving at p and the n^{th} wave which has suffered 2n reflections will be :

$$\delta_{2n} = \mu_2\,[pW_n + pW_o] + 2n\left(\frac{\lambda\beta}{2\pi}\right)$$

or

$$\delta_{2n} = \mu_2\{x_o[\cos(\theta_n-\varepsilon) - \cos(\theta_1-\varepsilon)] + y_o[\sin(\theta_n-\varepsilon) - \sin(\theta_1-\varepsilon)]\} + 2n\left(\frac{\lambda\beta}{2\pi}\right) \tag{1.4}$$

We may expand eq. (1.4) in powers of ε and retain terms only up to the second power. Then, we have :

$$\delta_{2n} = 2nt\mu_1\left\{1 - \frac{2n^2+3n+4}{3}\varepsilon^2 + \frac{\mu_1}{\mu_2}(n+1)\varepsilon^2\right\} - 2nX_o u_1 \varepsilon^2\left\{\frac{\mu_1}{\mu_2}(n+1) - 1\right\} + 2n\left(\frac{\lambda\beta}{2\pi}\right) \tag{1.5}$$

According to this equation the path difference is affected by the relative refractive index $\left(\frac{\mu_1}{\mu_2}\right)$ of the media inside and outside the interferometer.

I.2.2- Ray Tracing Method :

The fundamental phase condition could be derived by tracing the path of the rays [11].

I.2.2.a- The case of a silvered air wedge :

Consider a parallel beam of monochromatic light incident on a normal to one of the two components forming a silvered air wedge (Fig. 1.2.a). Take the apex of the wedge as the origin (o,o) and the vertical component as the y-axis. Multiply reflected beams after being transmitted intersect at point $P(X_o, Y_o)$. The total path of the beam after suffering n reflections is equal to :

$$\sum_{o}^{n-1} R_n + GP$$

From geometry of the figure, we have

$$R_o/R_1 = \cos 2\varepsilon/\cos_o\varepsilon$$

$$R_{n-1}/R_n = \cos (n+1)\varepsilon/\cos (n-1)\varepsilon$$

$$PH = (X_o - Y_o \tan\varepsilon) \cos\varepsilon/\cos (n+1)\varepsilon$$

$$R_n = X_o/\cos n\varepsilon - PH$$

The length of the path =

$$R_n\left\{\frac{\cos(n+1)\varepsilon}{\cos(n-1)\varepsilon} + \frac{\cos n\varepsilon \cos(n+1)\varepsilon}{\cos(n-2)\cos(n+1)\varepsilon} + \frac{\cos(n-1)\varepsilon \cos n\varepsilon \cos(n+1)\varepsilon}{\cos(n-3)\varepsilon \cos(n-2)\varepsilon \cos(n-1)\varepsilon} + \ldots\right\}$$

$$+ \frac{X}{\cos n\varepsilon}$$

$$= R_n \cos n\varepsilon \cos (n+1)\varepsilon \sum_{s=1}^{s=n} \frac{1}{\cos(s-1)\varepsilon \cos s\varepsilon} + \frac{X_o}{\cos n\varepsilon}$$

Now

$$\frac{\sin s\varepsilon}{\cos s\varepsilon \sin\varepsilon} - \frac{\sin (s-1)\varepsilon}{\cos(s-1)\varepsilon \sin\varepsilon} = \frac{\sin s\varepsilon \cos(s-1)\varepsilon - \sin(s-1)\varepsilon \cos s\varepsilon}{\cos s\varepsilon \cos (s-1)\varepsilon \sin\varepsilon} =$$

$$\frac{\sin[s\varepsilon - (s-1)\varepsilon]}{\cos s\varepsilon \cos(s-1)\varepsilon \sin\varepsilon} = \frac{\sin\varepsilon}{\cos s\varepsilon \cos (s-1)\varepsilon \sin\varepsilon} = \frac{1}{\cos (s-1)\varepsilon \cos\varepsilon}$$

and since,

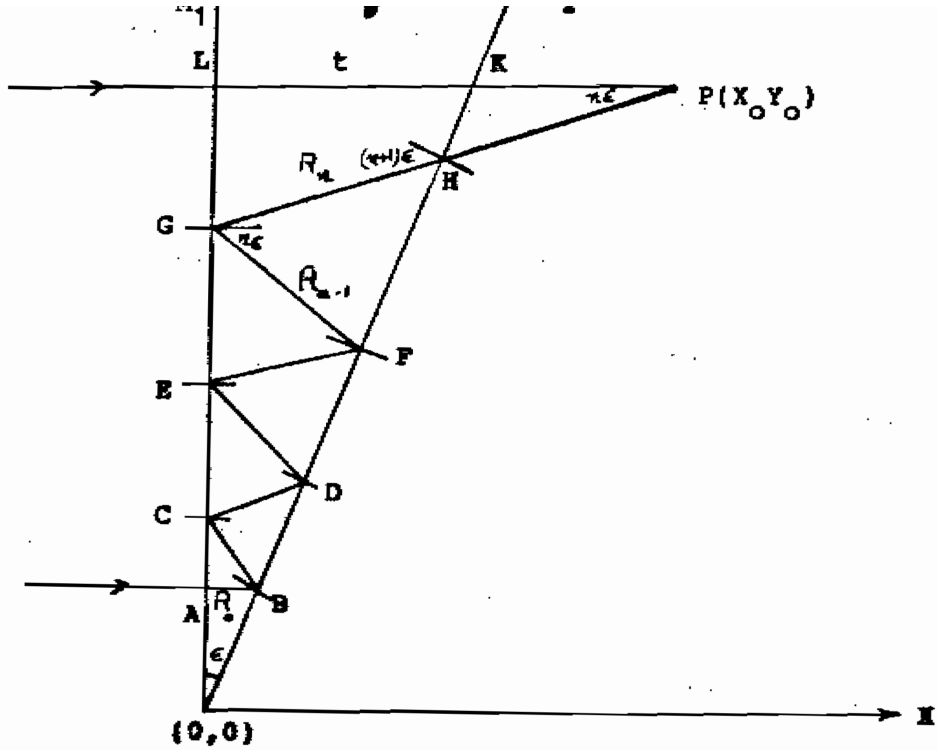

Fig. 1.2.a. Behaviour of multiple reflected beams in a silvered air wedge.

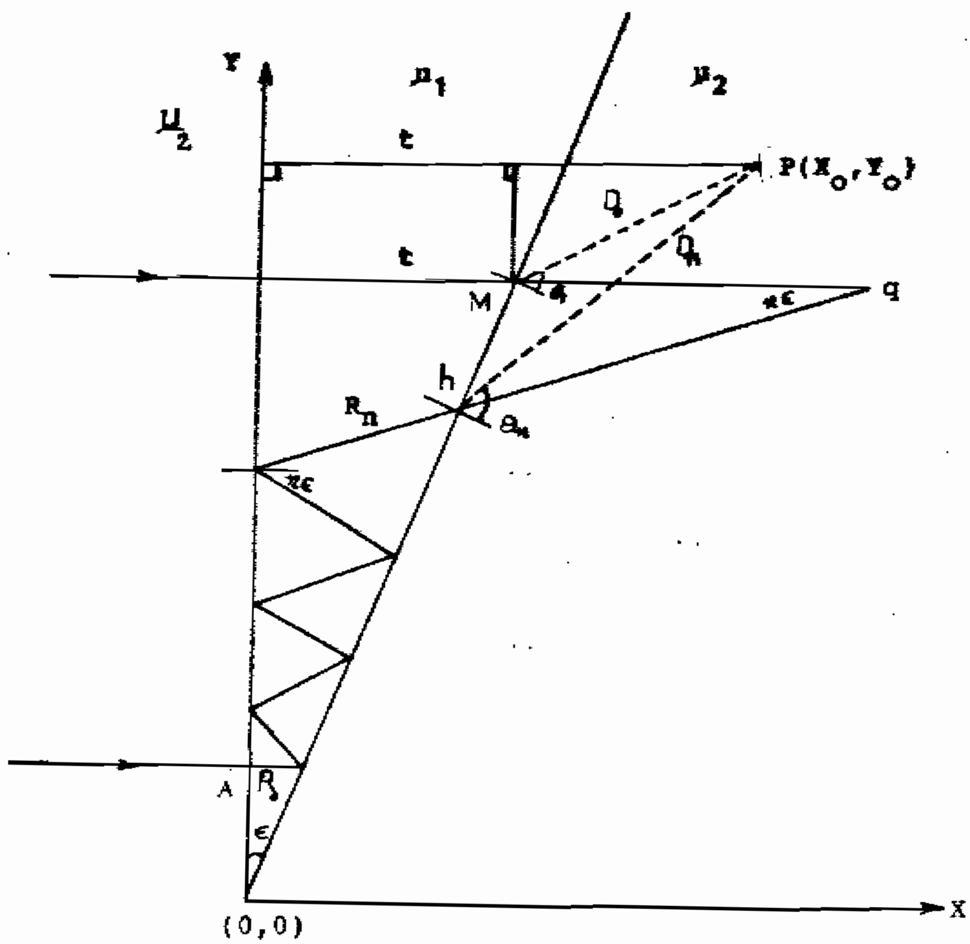

Fig. 1.2.b. Behaviour of multiple refelcted beams in a silvered wedge enclosing a dispersive medium.