

Ain shams University  
Faculty of Engineering  
Computer and Systems Department

MODEL REFERENCE ADAPTIVE CONTROL IN THE PRESENCE OF  
UNMODELED DYNAMICS AND SINUSOIDAL DISTURBANCES

by

ENG. ADEL TAWFIK YOUSSEF TAWFIK

A thesis submitted in  
fulfillment of the requirements of the degree of

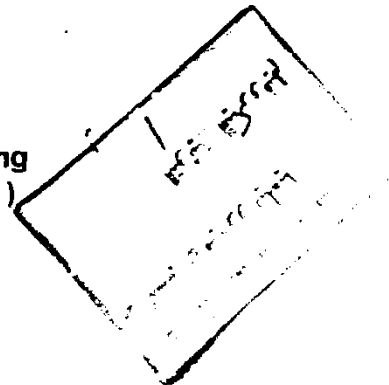
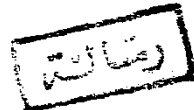
Ph.D.

in Electrical Engineering  
( Computer & Control )

Supervised by

Prof. Dr. A. M. WAHDAN

Cairo  
1993



£21.39

A.T

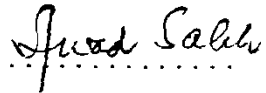
47213

EXAMINERS COMMITTEE

NAME, TITLE & AFFILIATION

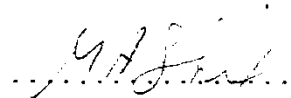
SIGNATURE

1. PROF. DR. A. I. SALEH

.....

Vice President of Assuit University, and  
Prof. of Automatic Control.

2. PROF. DR. M. A. SHEIRAH

.....

Head of Computer & Systems Engineering Dpt.,  
Faculty of Engineering, Ain Shams University.

3. PROF. DR. A. M. WAHDAN

.....

Prof. of Systems Engineering, Faculty of  
Engineering, Ain Shams University.

Date : 27 / 9 / 1993.



## STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Ph. D. in Electrical Engineering.

The work included in this thesis was carried out by the author, in the Department of Computer and Systems Engineering, Ain Shams University, from January 1991 to August 1993.

No part of this thesis has been submitted for a degree or qualification at any other university or institution.

Date : 27 / 9 / 1993.

Signature : 

Name : Adel Tawfik Youssef Tawfik

I wish to express my heartily gratitude to my parents for their loving attention, continuous encouragement and sincere prayers. First and foremost among those who helped me is my wife. She devoted every time and effort to keep me, not only dedicated for this research but also happy. With all my love, I thank her. My deepest love and thanks to my dear children who suffered a lot, during the long times, I have been taken from them. I only hope that I can make it up for them, in the future.

I thank all the teachers I have had through all years of formal education. I thank all my relatives, friends and colleagues who gave me any sort of support and concern.

\*\*\*\*\*

***TO THE SPIRIT  
OF MY MOTHER***

Title of thesis:

MODEL REFERENCE ADAPTIVE CONTROL IN THE PRESENCE OF  
UNMODELED DYNAMICS AND SINUSOIDAL DISTURBANCES

ABSTRACT:

This thesis is concerned with application of Model Reference Adaptive Control (MRAC) technique on real systems. After literature overviewing, it is assured that currently available standard adaptive control algorithms would likely become unstable in the presence of unmodeled plant dynamics and external sinusoidal disturbances. In such nonlinear dynamic control system, there exist two infinite gain operators, which can cause the loop gain to increase without bound; yielding an unstable control system as operation time proceeds.

For such problems to be tackled; we propose a modified MRAC-algorithm, that introduces a new concept of periodic resetting of the adaptation mechanism (PRAM). The optimal adaptation time used in the resetting mechanism, is computed on-line via minimizing an appropriate quadratic criterion.

A convergence test is proposed to detect early, any expected instability; in order to switch on the PRAM-algorithm in the proper time.

For realizing best possible performance, we proposed a new simple procedure to determine on-line the proper vector of gain parameters, in a near-optimal way. The obtained values are used for initializing the PRAM-algorithm.

An illustrative example, chosen from literature, is used to test the validity of the analytical arguments and improvements of the proposed algorithm. The obtained simulation results are quite satisfactory.

The developed technique has been also applied to the longitudinal control problem of a high-speed fighter aircraft, in the presence of unmodeled dynamics and sinusoidal disturbances. Simulation results have proved that the proposed control technique is robust, and capable of solving problems that face the other available techniques.

## SYMBOLS AND ABBREVIATIONS

$A(q^{-1})$	denominator polynomial of a discrete transfer function in the backward shift operator $q^{-1}$ .
$B(q^{-1})$	numerator polynomial of a discrete transfer function in the backward shift operator $q^{-1}$ .
$A^{(n)}$	indicates that the polynomial $A(q^{-1})$ has degree $n$ .
$B^{(m)}$	indicates that the polynomial $B(q^{-1})$ has degree $m$ .
$n^*$	indicates relative degree, $n^* = n - m$ .
$s$	Laplace transform variable in continuous transfer functions, unless it is stated that $s = \frac{d}{dt}$ (as time-derivative operator).
$g_p$	plant gain, in plant transfer function.
$y_p(t)$	plant output.
$y(t)$	output of the system.
$u(t)$	control signal, or manipulated input to the controlled
$y_m(t)$	output of the reference model.
$r(t)$	reference command, or reference input (set point).
$e(t)$	output error signal, $e(t) = y(t) - y_m(t)$ .
$d_p$	delay of the plant.
$d$	delay of the system.
MRAC	Model Reference Adaptive Control.
ROMRAC	Reduced Order Model Reference Adaptive Control.
CA1	Continuous Adaptive Algorithm No. 1.
DA2	Discrete Adaptive Algorithm No. 2.
$d(t)$	output disturbance.
$d_o$	amplitude of sinusoidal disturbance.
$\omega_o$	frequency of sinusoidal disturbance, $d(t) = d_o \sin \omega_o t$ .

PRTF	Positive Real Transfer Function.
LTI	Linear Time Invariant.
$\Gamma$	matrix of adaptation gain coefficients $\gamma_i$ , or adaptation rate factors.
$\underline{W}$	vector of auxiliary variables, used in the structure of adaptive algorithms (e.g. CA1, DA2).
$\underline{K}$	vector of gain parameters.
$\underline{K}^*$	vector of nominal gain parameters.
$\tau$	indicates time constant .
$\tau_m$	time constant of the reference model.
$\  \cdot \ $	norm of a function, or a vector.
$t$	time
$T_s$	sampling-period of the discrete system.
$t_s$	settling-time of system's transient response.
$T_a$	adaptation-time.
$T_{a_{opt}}$	optimal choice for adaptation-time.
$\sum$	summation.
$\oplus$	adder.
$\otimes$	multiplier
CT	Convergence Test.
$\Delta T$	time-interval, every which CT is periodically operated.
$V$	performance criterion.
SV	Slope of $V$ , or performance gradient ( $V'$ ).
$\omega_n$	natural frequency of a second order system.
$\omega_d$	damped frequency of a second order system.
$\xi$	damping coefficient of a second order system.
$t_{P_1}$	time till first overshoot.
$\Delta_1$	first overshoot ratio.

$t_{p_2}$	time till second overshoot.
$\Delta_2$	second overshoot ratio.
PRAM	Periodic Resetting of Adaptation Mechanism.
IG's	Initial Gains.
IGAA	Initial Gains Adjustment Algorithm.
TCYC	the cyclic period of resetting the PRAM- algorithm, (TCYC = $T_{a_{opt}}$ ).
J	no. of computation cycle in the on-line adaptive control.
DF	Disturbance-Free.
$\delta_r(K_i)$	absolute relative change of gain parameter $K_i$ .
$\delta_r(V)$	absolute relative change of performance V.
$\theta$	the change in pitch angle of aircraft.
$\theta_r$	Aircraft reference input of pitch (set point).
sp	indicates short-period mode of an aircraft, in the longitudinal vertical plane of motion.
ph	indicates phugoid mode of an aircraft, in the longitudinal vertical plane of motion.
$\alpha$	the change in aircraft angle of attack.
$\delta_e$	the change in elevator deflection angle.

## LIST OF FIGURES AND TABLES

	page
Fig.2.1: Controller Structure of CA1 with Additive Output Disturbance $d(t)$ . ....	16
Fig.2.2: Error System for CA1. ....	21
Fig.2.3: Controller Structure for DA2. ....	26
Fig.2.4: Error System for DA2. ....	28
Fig.3.1: Pole Trajectories of Model Optimal Control in the S-Plane as the Control Weight is Decreased. ....	40
Fig.3.2: Error System for CA1. ....	41
Fig.3.3: Block Diagram of a Generic Adaptive Controller. .	46
Fig.3.4: Error System of CA1 emphasizing role of $G_w(t)$ . ..	50
Fig.3.5: Modified Adaptive Parameters $\tilde{K}_m(t)$ using PRAM with CA1. ....	59
Fig.3.6: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and $d(t)=0.001 \sin 5t$ . ....	67
Fig.3.7: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and $d(t)=0.001 \sin 6t$ . ....	68
Fig.3.8: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and $d(t)=0.001 \sin 10t$ . ....	69
Fig.3.9: Simulation of Output Response for CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and (a) $d(t)=0.001 \sin 5t$ . (b) $d(t)=0.0004 \sin 5t$ . (c) $d(t)=0.00001 \sin 5t$ . ....	70
Fig.3.10: Simulation of Control Signal for $\gamma=4$ , $r(t)=0.3$ , and (a) $d(t)=0.001 \sin 5t$ . (b) $d(t)=0.0004 \sin 5t$ . (c) $d(t)=0.00001 \sin 5t$ . ....	71

Fig.3.11: Simulation of Parameters for CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and	
(a) $d(t)=0.001 \sin 5t$ .	
(b) $d(t)=0.0004 \sin 5t$ .	
(c) $d(t)=0.00001 \sin 5t$ . ....	72
Fig.3.12: Simulation of Output Response for CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t) = 0.3$ and $d(t)=1 \times 10^{-6} \sin 20t$ .	
(a) Without PRAM-Modification.	
(b) With PRAM-Modification, using $T_a=2$ sec. ...	73
Fig.3.13: Simulation of Parameters for CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and $d(t)=1 \times 10^{-6} \sin 20t$ .	
(a) Without PRAM-Modification.	
(b) With PRAM-Modification, using $T_a=2$ sec. ....	74
Fig.3.14: Simulation of Control Signal for CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ and $d(t)=1 \times 10^{-6} \sin 20t$ .	
(a) Without PRAM-Modification.	
(b) With PRAM-Modification, using $T_a=2$ sec. ....	75
Fig.3.15: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ , $d(t)=1 \times 10^{-6} \sin 5t$ , using PRAM-Modification, with $T_a=1.8$ sec. ....	76
Fig.3.16: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ , $d(t)=1 \times 10^{-6} \sin 5t$ , using PRAM-Modification, with $T_a=1.4$ sec. ....	77
Fig.3.17: Simulation of CA1 with Unmodeled Dynamics, $\gamma=4$ , $r(t)=0.3$ , $d(t)=1 \times 10^{-6} \sin 5t$ , using PRAM-Modification, with $T_a=4$ sec. ....	78
Fig.4.1: $T_{a_{opt}}$ is at Time of Conversion of $V^{\wedge}$ direction from Negative to Positive, after Transient is over. ....	88
Fig.4.2: Time-Response of An Adaptive System with a step Reference Input $r(t)$ . ....	94
Fig.4.3: Flow-Chart of Adaptive Algorithm DA2 with Proposed PRAM-Modification, using Conventional Sequential Processing. ....	112

Table 4.1: Values of Performance Criterion for Different Simulations. ....	117
Fig.4.4: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma=0.5$ , $k_y(0) = -0.65$ , $k_r(0) = 1.14$ and $d(t)=0.02 \sin 10t$ . ....	122
Fig.4.5: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma=0.5$ , $k_y(0) = -0.65$ , $k_r(0) = 1.14$ and $d(t)= 0.05 \sin 10t$ . ....	123
Fig.4.6: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma= 0.5$ , $k_y(0) = -0.65$ , $k_r(0)=1.14$ and $d(t)=0.10 \sin 10t$ . ....	124
Fig.4.7: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma= 0.5$ , $k_y(0) = -0.65$ , $k_r(0) = 1.14$ and $d(t)= 0.10 \sin 4t$ . ....	125
Fig.4.8: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma= 0.5$ , $k_y(0) = -0.65$ , $k_r(0)= 1.14$ and $d(t)= 0.10 \sin 6t$ . ....	126
Fig.4.9: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma= 0.5$ , $k_y(0) = -0.65$ , $k_r(0) = 1.14$ and $d(t)= 0.10 \sin 8t$ . ....	127
Fig.4.10: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma= 0.8$ , $k_y(0) = -2.5$ , $k_r(0)= 3.6$ and $d(t)=0$ . ....	128
Fig.4.11: Simulation of DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma=0.8$ , $k_y(0)=-2.5$ , $k_r(0)=3.6$ and $d(t)=0.05 \sin 5t$ . (a) Outputs ( $Y, Y_m$ ) and Parameters ( $k_r, k_y$ ). (b) Performance ( $V$ ) and Performance Gradient ( $SV$ ). ....	129
Fig.4.12: Simulation of Output Response for DA2 with Unmodeled Dynamics, $r(t)= 1$ , $\gamma=0.8$ , $k_y(0) = -2.5$ , $k_r(0)=3.6$ and $d(t)= 0.05 \sin 5t$ . (a) Without PRAM-Modification. (b) With PRAM-Modification, with $T_{a_{opt}} = 2.64$ sec. ....	131
Fig.4.13: Simulation of Parameters for DA2 with Unmodeled Dynamics, $r(t)= 1$ , $\gamma=0.8$ , $k_y(0) = -2.5$ , $k_r(0)=3.6$ and $d(t) = 0.05 \sin 5t$ . (a) Without PRAM-Modification. (b) With PRAM-Modification, with $T_{a_{opt}} = 2.64$ ....	132

- Fig.4.14: Simulation of Control Signal for DA2 with Unmodeled Dynamics,  $r(t)=1$ ,  $\gamma=0.8$ ,  $k_y(0)=-2.5$ ,  $k_r(0)=3.6$  and  $d(t)=0.05 \sin 5t$ .  
 (a) Without PRAM-Modification.  
 (b) With PRAM-Modification, with  $T_{a_{opt}} = 2.64$  sec. 133
- Fig.4.15: Simulation of Performance Criterion for DA2 with Unmodeled Dynamics,  $r(t)=1$ ,  $\gamma=0.8$ ,  $k_y(0)=-2.5$ ,  $k_r(0)=3.6$  and  $d(t)=0.05 \sin 5t$ .  
 (a) Without PRAM-Modification.  
 (b) With PRAM-Modification, with  $T_{a_{opt}} = 2.64$  sec. 134
- Fig.4.16: Simulation of DA2 with Unmodeled Dynamics, with Switching PRAM - Modification at Instability Prediction by CT,  $r(t)=1$ ,  $\gamma=0.5$ ,  $k_y(0)=-0.65$ ,  $k_r(0)=1.14$ , and  $d(t)=0.02 \sin 10t$ .  
 (a) Outputs ( $Y, Y_m$ ) and Parameters ( $k_r, k_y$ ).  
 (b) Control Signal( $U$ ) and Performance Criterion( $V$ ) 135
- Fig.4.17: Simulation of DA2 with Unmodeled Dynamics, using Suboptimal Initial Gains,  $r(t)=1$ ,  $\gamma=0.5$ ,  $k_y(0)=-3.88$ ,  $k_r(0)=5.01$  and  $d(t)=0$ . ..... 137
- Fig.4.18: Simulation of Output Response for DA2 with Unmodeled Dynamics,  $r(t)=1$ ,  $\gamma=0.5$ ,  $d(t)=0$ , using Different Initial Gains:  
 (a) Intuitive IG's,  $k_y(0) = k_r(0) = 5$ .  
 (b) Zero IG's,  $k_y(0) = k_r(0) = 0$ .  
 (c) Suboptimal IG's,  $k_y(0)=-3.88$ ,  $k_r(0)=5.01$  .. 138
- Fig.4.19: Simulation of Parameters for DA2 with Unmodeled Dynamics,  $r(t)=1$ ,  $\gamma=0.5$ ,  $d(t)=0$ , using Different Initial Gains:  
 (a) Intuitive IG's,  $k_y(0)=k_r(0)=5$ .  
 (b) Zero IG's,  $k_y(0)=k_r(0)=0$ .  
 (c) Suboptimal IG's,  $k_y(0)=-3.88$ ,  $k_r(0)=5.01$ . 139
- Fig.4.20: Simulation of Control Signal for DA2 with Unmodeled Dynamics,  $r(t)=1$ ,  $\gamma=0.5$ ,  $d(t)=0$ , using Different Initial Gains:  
 (a) Intuitive IG's,  $k(0)=k(0)=5$ .

(b) Zero IG's, $k(0) = k_r(0) = 0$ .	
(c) Suboptimal IG's, $k_y(0) = -3.88$ , $k_r(0) = 5.01$ .	140
Fig.4.21: Simulation of Performance Criterion for DA2 with Unmodeled Dynamics, $r(t)=1$ , $\gamma=0.5$ , $d(t)=0$ , using Different Initial Gains:	
(a) Intuitive IG's, $k_y(0) = k_r(0) = 5$ .	
(b) Zero IG's, $k_y(0) = k_r(0) = 0$ .	
(c) Suboptimal IG's, $k_y(0) = -3.88$ , $k_r(0) = 5.01$ .	141
Fig.5.1: S, b, and c of Aircraft wing, Shaded Area = S.	147
Table 5.1 Values of Short - Period Mode Parameters for Different Altitudes of Aircraft. ....	149
Fig.5.2: Block diagram of the Existing Aircraft Autopilot System (pitch-channel). ....	150
Fig.5.3: Simulation of DA2 with First Order Model and Intuitive Choice of Initial Gains, $r(t)=0.01$ , $\gamma=0.4$ , $d(t)=0$ , $k_y(0)=-0.5$ and $k_r(0)=1$ .	
(a) Output and Parameters ( $Y, Y_m$ and $k_y, k_r$ ).	
(b) Control and Performance Criterion (U and V).	167
Fig.5.4: Simulation of DA2 with First Order Model, $r(t) = 0.01$ , $d(t)=0.001 \sin 5t$ , $\gamma=0.4$ , $k_y(0)=-0.5$ and $k_r(0)=1$ .	
(a) Output and Parameters ( $Y, Y_m$ and $k_y, k_r$ ), for TEND= 97.3 sec.	
(b) Output and Control ( $Y, Y_m$ and U), for TEND=10sec.	
(c) Performance Criterion and Performance Gradient (V and SV), for TEND = 10 sec. ....	169
Fig.5.5: Simulation of DA2 with First Order Model, $r(t)=0.01$ , $\gamma=0.4$ , $d(t)=0.002 \sin 5t$ , $k_y(0) = -0.5$ and $k_r(0)=1$ . ....	172
Fig.5.6: Sumulation of DA2 with First Order Model, $r(t)=0.01$ , $\gamma=0.4$ , $d(t)=0.004 \sin 5t$ , $k_y(0)=-0.5$ and $k_r(0)=1$ . ....	173