Developments In The Cumulative Sum Control Chart Technique

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CONTENTS

I Introduction

W. A. Shewhart was the first to apply the statistical methods to the problem of quality control. He made the first proposal for a modern control chart (37).
On this chart, results of individual samples are plotted and rules are given to enable an interpretation to be made easily. According to the original proposal the chart is furnished with what is called "action lines," so that when any point falls outside these lines, some action is to be called for. Clearly, with such a simple rule, the samples are actually classed as either good or bad; the good ones whose points fall within the action lines, suggest that the process should continue without special attention or interruption, while the bad ones with points outside the lines demand a corrective action.

One of the decisions to be taken, in constructing a Shewhart control chart, is the position of the action lines. Usually three sigma limits are set up. The average number of points plotted on the chart, before a signal of out of control is given, is known as the average run length(ARL). This statistics depends on the amount of shift from the target value, as shown in table(1), for observations of normal distribution. The data of table (1) show that as the amount of shift increases, the ARL decreases, which may indicate that larger shifts in the process average will be detected guicker.

If one wants to improve the sensitivity of the control chart to small amounts of shifts, this can be effected by narrowing the region between the action limits. Thus if the resulting amount of shifts is equal to 2σ then the ARL =6.3 when 3σ-limits are used and the ARL = 2 if 2σ-limits are employed. However this procedure of narrowing the control limits, results in more frequent false alarms when the process is in control, as indicated by the data of the first row of table (1). These data show that on the average 385 points will be plotted before a false alarm for out of control is given in case 3 σ limits are used on the chart, while search for such out of control will take place after every 22 points are plotted when 2σ control limits are used.

Because of this disadvantage of the Shewhart chart to detect quickly small shifts in the process level, several modifications of it have been introduced. All these modifications make use of the information provided by several numbers of the previous points. The simplest modification is the introduction of so-called warning limits in addition to the action limits (30). Traditionally the warning limits are taken at 2σ with the action limits at 3σ . The rule for operating the control chart in this case is :-

" Whenever either a single observation lies outside the action limits or two successive observations lie between the warning and action limits (at the same side of the average), an out of control single is given ".

Table (1) Average Rum Lengths of Shewhart chart applied to normally distributed variates.

Shift(in sigma)	ARL for 3σ limits	ARL for 2σ limits
0.0	385.	22.0
0.4	200.	15.9
0.8	71.4	8.50
1.2	27.8	4.70
1.6	12.4	2.90
2.0	6.30	2.00
2.4	3.65	1.52
2.8	2.38	1.27
3.2	1.73	1.13
3.6	1.38	1.058
4.0	1.19	1.023
5.0	1.023	1.0013

This sort of scheme has a steeper average run length function when plotted against the size of the departure from target value than the original Shewhart scheme. This is illustrated in table (2), in which the ARL's for the modified chart are shown. The values of the limits have been set at 3.1 and 2.1 sigma for the action and warning limits respectively, to give approximately the same ARL at zero bias.

When comparing the values of A R L of table (2) with the corresponding values of table (1) for the unmodified Shewhart chart, we notice that the unmodified chart reacts more quickly than the modified chart only for a large bias (larger than 4 sigma). This is because the action limits in the later had to be chosen slightly higher (3.1 σ) than those of the shewhart chart to give almost the same average number of false alarms.

Table (2). A R L for a Shewhart chart with warning limits for normal variates

Shift (in sigma)	A R L
0.0	390
0.4	187
0.8	57.2
1.2	20.16
1.6	8.78
2.0	4.65
2.4	2.90
2.8	2.06
3.2	1.61
3.6	1.35
4.0	1.195
5.0	1.029

Some other rules use runs of points on the control chart. For example, Moore (26) and Weiler (45) set rules of the type " If K consecutive points on the chart fall outside warning limits take action".

Such rules were successful in deterting genuine shifts in the process level, bacause they take into account part of the informations contained in the chart for a fixed number of samples in the immediate past.

Trials were then made to relaxe the restriction that only a fixed number of past samples should be considered in taking a decision. The cumulative sum charts do posses this property and use the combined information of all the observations that have been obtained up to the time of testing. Also they take into account the actual positions of the points on the chart and not merely the classification into which division of the chart the points falls.

Also, by plotting the cumulative sum of the observations, instead of the individual observations, one can see by a glance not only the value of the last observation, corresponding to the slope of the line joining the current point to the point immediately preceeding it, but also the mean of the last two observations which correspounds to the slope of the line joining the current point to the last point but one. Similarly one can see, at the same time, the mean of the last three, the last four and the last N observations for any N.

As to sensitivity of cusum chart compared with that of the standard Shewhart chart, it is to be noticed that for moderate shifts, the cusum chart is more sensitive than the Shewhart chart. For example, considering a standard chart with fixed control limits at ($\pm 3\sigma$) controlling the mean of a normal population of constant variance, σ^2, there is an average run length of about 44 single observations before a shift in the process mean of one standard deviation from the target value is detected . But the ARL, in this particular case, using the cusum instead of the shewhart one is equal to 13.2 observations, with a reduction factor equal to about 70 % in the amount of samples inspected, giving us quicker detection power than the Shewhart chart.

Also, this new technique enables us to evaluate the process shift magnitude which depends on the ability to estimate the slope of the line joining the "out of control" points.

Moreover, On the cumulative chart, the points at which process changes occur stand-out vividly, however, such changes are largely hidden when using the standard Shewhart chart.

At last, it is worth to notice that the cusum chart presents a clearer picture of the true system behaviour as it resolves the shotgun or scattered appearance of the regular control chart into a visual tracking of process changes.

As a simple illustration, the cusum for the mean of an industrial process, is developed by plotting the cumulative sum $S_m = \sum_{i=1}^{m} (X_i - \mu)$ vs. i. So long the process mean remains near the target value, the graph of the cumulative sum should not deviate too much from the horizontal line. To check large deviations, a V-shaped mask, for instance, is placed on the chart with the vertex of the V pointing horizontally forwards and at a horizontal distance d ahead of the last plotted point on the chart.
If the process is under control, then all the curve remains visible, while if the process mean has fallen below or above the target, the curve will disappear under the upper or lower limb of the V-mask respectively. The two parameters d and θ defining the applied V-mask, are determined by the control procedure.

Inspite of the many advantages of the cusum charts, they exhibit certain disadvantages. First, Shewhart charts are faster than the cusum ones in detecting large shifts of the process mean. Secondly, on a Shewhart chart there are visible limits which point out vividly the out of control conditions, the cusum chart lacks this property, a mask is to be used to indicate the states of control and out of control. Besides, the mask dimensions are not easy to determine, they require certain mathematical techniques which are usually beyond the ability of the ordinary personnel. More specifically, the determination of the mask dimensions needs the evaluation of the behaviour of ARL function with respect to the magnitude of the shift in the process mean. The equations which describe this behaviour is of the type " inhomogenous Fredholm equations " of the second kind (42) . The solution of such types of equations needs certain mathematical techniques which are troublesome in nature, especially when the distribution is not normal. Thus, for discrete distribution such as binomial or Poisson, it is necessary to formulate and solve sets of equations to which we have already referred.

Since the advent of the digital computer in the early 1950's, a new and powerful approach have been created in dealing with such problems. The new approach is known as the simulation technique which involves setting up a model of a real situation with parameters equal to the required unknown quantities, and then experiments are performed on the model. The unknowns are determined by computing the corresponding parameters in the constructed model which are approximately equal to the required parameters.

Goldsmith and Whitefield (10) adopted this technique in constructing the cusum chart for the case of normal distribution only. Since then the simulation technique have never been applied in the Cusum charts. One main task for preparing this thesis is to construct cusum charts of non- normal distribution using the simulation technique.

The simulation technique is described at large in chapter 5. In chapter 4 are described the methods for generating stochastic variates needed for the simulation programmes. A full discussion for generating uniformly distributed pseudo random variates is also given which play the major role in generating random variates drawn from the other probability distribution. A generator is devised for the special case where the bit word size of the computer is equal to 48. This generator is adequately suitable for running on the I.C.L. 1905 Electronic Computer available at Cairo university . Chapler 2 is devoted for the review of literature and chapter 3 for discussing the various analytic approaches of the problem.

The other point, believed to be of importance in association with the construction of a cusum chart, is that of considering the economical factor in the design. Taylor (39) and sultan (41) have tackled the economical design by maximizing the long run time average net income per cycle.

Each of the two authors dealt with normal variates only. In their