HYDROMAGNETIC STABILITY OF A CYLINDRICAL SURFACE

A Thesis
Submitted in Partial
Fulfillment of the Requirements
for the Award of the Master Degree
in Science (Applied Mathematics)

By

AZIZA ABD EL MENEIM DEMERDASH

From
Women's University College
Ain Shams University
Cairo

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بسم الله الرحمن الرحيم

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سبحانك لا علم لنا إلا ما علمتنا إنك أنت العليم الحكيم

صدق الله العظيم

سورة البقرة ٣٢

Ain Shams University
University College for Women
Mathematics Department

M.Sc. Thesis (Mathematics)

Title of Thesis:

HYDROMAGNETIC STABILITY

OF A CYLINDRICAL SURFACE

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HYDROMAGNETIC STABILITY OF A CYLINDRICAL SURFACE

Besides the work carried out in this thesis, the candidate has attended and passed the following post-graduate courses.

1- NUMERICAL ANALYSIS.

2- MATHEMATICAL ANALYSIS 1.

3- MATHEMATICAL ANALYSIS 2.

Head of Mathematics Department

PRESENTED

TO

MY HUSBAND

ACKNOWLEDGEMENTS

I am deeply grateful to Professor Dr. F.A. Ayoub Professor of Applied Mathematics, University College for Women, Ain Shams University for his advices and encouragements.

I am greatly indebted to Dr. S.S. ELAZAB Assistant Professor of Applied Mathematics, University College for Women, Ain Shams University for suggesting the problem in this work and tackling their difficults. Moreover, I thank her also for her helpping from the beginning till the end of this thesis.

I like also to thank Dr. A.E. RADWAN, Department of Mathematics, Faculty of Science, Ain Shams University for his kindeness and his fruitful discussions throughout my work.

Finally my sincere thanks to Head and staff-members of the MATHEMATICS DEPARTMENT in AIN SHAMS UNIVERSITY, University College for Women, for their encouragement.

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ARABIC SUMMARY..

SUMMARY

SUMMERY

The thesis is concerned with the magnetohydrodynamic stability of various cylindrical surfaces under the influence of surface tension and pervaded by an axial magnetic field in the direction of the axis of the cylinder.

In chapter "I" we introduce the subject of magneto-hydrodynamics. We explain the fundamental MHD equations of motion in this field. Also we discuss the stability concepts and the techniques followed in this respect such as the normal modes analysis to the perturbation equations. On the other hand we discuss some boundary conditions which we used in this thesis. We have studied the previous studies concerning the work of this thesis.

In chapter "II" we have studied the stability of a liquid cylinder of radius R_o and density ρ under the influence of surface tension and pervaded by a uniform axial magnetic field $(0,0, H_o)$. We have deduced the dispersion relation of the liquid cylinder and it is found that for non-axisymmetric modes (i.e.$m \geq 1$), both the two forces are stabilizing for all short and long wavelengths.

In the axisymmetric mode $m=0$ we have deduced that the capillary force is stabilizing if $1 \leq x < \infty$ ($x=kR_o$,

$k = \frac{2\pi}{\lambda}$ is the longitudinal wave number), while it is destabilizing as long as $0 \le x < 1$ and the electromagnetic force is stabilizing for all values of $x \neq 0$ (i.e. for all short and long wavelengths). In the later case we have deduced that this model is MHD stable in the $m \geq 1$ for $x \neq 0$, while in the $m=0$ it is destabilizing for small wavelength.

In chapter "III" we have studied the stability of a streaming (velocity $U\underline{e}z$) fluid cylinder of radius R_o and density ρ immersed in a streaming finite liquid cylinder of radius qR_o, $1<q<\infty$ and density ρ' for all axisymmetric and non-axisymmetric perturbations. The system is affected by surface tension and axial magnetic field. The dispersion relation is derived and discussed. The analyticall results are confirmed numerically and it is found that q is a factor of strongly destabilizing influence even for large values of (H_o/H_s), $H_s=(T/\mu R_o)^{\frac{1}{2}}$ or/and (ρ'/ρ).
For the same values of q and ρ'/ρ values, it is found that the magnetic field is strongly stabilizing effect.
For the same values (H_o/H_s) and q values, it is found that the domains of instability are slightly decreasing with increasing $(\frac{\rho'}{\rho})$ values.

CHAPTER I

CHAPTER (I)

INTRODUCTION

(I.1) MAGNETOHYDRODYNAMICS (MHD):

MHD is the study of the macroscopic interaction of electrically conducting liquids and gases in the presence of magnetic fields. The necessary properties of fluid dynamics will be developed directly from the basic conservation laws of mass, momentum and energy.
MHD is relatively new for some extent and important pranch of fluid dynamics due its applications in science.

The word fluid will be used here, as usual to denote a continuummedium which may be either a liquid or a gas.

If a magnetized medium moves, an electric field is produced. If the medium is electrically conducting and different parts of it moves at different velocities, the electric field will produce currents. These currents interact with the magnetic field and produce forces which are strong enough to change the state of motion of the medium appricably.
In this way the hydrodynamic motion and electromagnetic phenomena are coupled. So (MHD equations) express the coupling between the hydrodynamic equations and those of the electromagnetic theory.