GENERALIZED FIELD THEORIES

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M. Sc. COURSES

STUDIED BY THE AUTHOR (FEB. 1981 - FEB. 1982)

(AT AIN SHAMS UNIVERSITY)

(i) Relativity

2 hours weekly.

(ii) Solid State Physics I

2 hours weekly.

(iii) Solid State Physics II

2 hours weekly.

(iv) Applied Numerical Analysis

2 hours weekly.

(v) Analytical dynamics

2 hours weekly.

(vi) Electromagnetism

2 hours weekly.

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PREFACE

The thesis started by giving a brief account of the four fundamental interactions responsible of all the microscopic and macroscopic phenomena occurring in the universe. It shows the need for a generalized field theory capable of describing all or some of these interactions in a unified way. some of the important ways to generalize the existing field theories are discussed and in particular those using **absolute parallelism geometry (APG)**.

More strictly, the thesis studies, in some details, the most recent of these generalized field theories - the theory estabilished by Hayward in 1981. It is one of the theories using **(APG)**, as a geometrical structure, in their formalism. Haywards theory is then applied to two cases of physical interest and some satisfactory results are obtained.

The thesis comparises five chapters, the first of which gives a preliminary introduction explaining the different fundamental interactions in the universe. It discusses the need for a generalized field theory and the different approaches to do so.

The second chapter deals briefly with the previous generalized field theories based on (APG) in their formalism. A brief discussion of the method of deriving each theory is given, and the results obtained, if any.

In chapter III, a detailed discussion of Hayward's theory is given with a complete re-derivation of its field equations. The P.P.N. solution of the field equations is obtained and thus the numerical values of the constants involved are determined.

In chapter IV, the application of Hayward's theory to the two particular cases of physical interest:

(a) the case of a static spherically symmetric space,

(b) the case of the istropic homogeneous space used in the cosmological problem,

is given. Some results of physical interest are obtained. This constitutes the original part in the present study.

Finally, chapter V comprises some concluding remarks and the comparison of the results obtained by applying Hayward's theory with those derived from other field theories.

Chapter "I": INTRODUCTION.

I.1: The Fundamental Fields in the Universe.

A) Gravity.

B) The Electromagnetic Field.

C) The Strong Field.

D) The Weak Field.

I.2: Need for a Generalized Field Theory.

I.3: Ways of Generalization.

A) Quantization.

B) Geometrization.

I.4: Geometrization and Generalized Field Theories.

A) Generalization Using Riemannian Spaces.

B) Generalization Using Non-Riemannian Spaces.

I.5: Aim of the Present Work.

I.1 THE FUNDAMENTAL FIELDS IN THE UNIVERSE.*

Observed phenomena in the universe can be classified into two main types:

(i) Macroscopic phenomena concerned with galaxies, stars, planets and other macroscopic bodies (i.e. bodies of non-negligible dimensions.).

(ii) Microscopic phenomena concerened with microscopic particles (i.e elementary particles.).

Elementary particles are those which have no structure i.e they don't consist of other particles, they can not split into other parts. Besides the three structural particles of the atom (electron, proton, neutron), at the present time, about 200 more elementary particles are known. Most of them are unstable as they soon decay spontaneously, after being formed due to collision with other particles, transforming into more stable particles. The life time of such aprticles is very short and ranges from 10^{-22} to 10^{-6}s.

There are nine stable elementary particles known so far. These are the photon γ , 4 neutrinos $\nu_\mu, \nu_e, \bar{\nu}_\mu, \bar{\nu}_e$, 2 electrons e^-, e^+ and 2 nucleons p, $\bar{p}$. Except for the photon and the neutral pi-meson (π°-meson), any elementary particle exists in two different forms one of which is an antiparticle

* **For more details we refer to the text books:**
  (i) The Ideas of Particle Physics, J.E.Dodd, Cambridge University Press (1984).
  (ii) Particles, Quanta & Waves, Ya.A.Smorodinsky, Mir Publishers, Moscow (1976).

relative to the other In collisions of a particle and its antiparticle, the two particles disappear and new particles with lower rest mass appear or energy is released.

Both elementary and compound particles have either half-integer (i.e 1/2, 3/2, 5/2, 7/2 , ...) or integer (i.e 0,1,2,3,.....) spin. Particles having half-integer spin obey Pauli's principle which don't allow two similar particles to be found in identical states described by the same set of quantum dynamic values. Accordingly, such particles, for example electrons, neutrons and protons, are distributed in shells. Only elementary particles with the half-spin can form a substance. Such particles are described by the Fermi-Dirac statistics and are called Fermions. Particles with integer spin don't obey Pauli's principle. They have a tendency to accumulate in a single quantum state with the lowest energy. These particles are described by Bose-Einstein statistics and are called Bosons. Bosons can be interaction carriers(i.e field quanta).

Elementary particles have the ability to interconvert by collisions and thus experience spontaneous decaying or annihilation. The conversion of elementary particles is conserved with respect to the total energy, total momentum, total angular momentum and total charge and obeys certain rules discovered experimentally.

In elementary particle interactions, gravity is almost negligible. The strength of the interaction is estimated by the value of the interaction constant. The value of this constant is determined experimentally. The mean time interval during which the phenonena are developing under the influence of the interaction is known as the characteristic time. The shorter the characteristic time of the interaction is, the more its strength will be.

The phenomena observed in the universe can be attributed to the effects of four fundamental types of interaction (fields) whcih are:

(i) The gravitational field (with characteristic time of order 10^{16}s.)

(ii) The weak field (with characteristic time of order 10^{-10} s.).

(iii) The electromagnetic field (with characteristic time of order 10^{-21} s.).

(iv) The strong field (with characteristic time of order 10^{-23}s.).

This means that the gravitational field is the weakest of the four and the strong field is the strongest one.

Macro-world phenomena are mainly attributed to the gravitational and electromagnetic fields. This is due to the fact that they are the only interactions that have significant effects at observable ranges (i.e they have a very long range). The effects of the weak and strong fields are only confined within ranges of order 10^{-13} cm at most.

On the other hand, microscopic phenomena are due to the electromagntic, weak and strong fields. The following table may give an idea about the difference between the four fundamental fields in Nature:-

Field	Range (in cm.)	Characteristic (in seconds)	Acts on:
Gravity	$\longrightarrow \infty$	10^{16} s.	All particles.
Weak Interaction	$\sim 10^{-16}$ cm.	10^{-10} s.	Leptons and hadrons.
Electromagnetism	$\longrightarrow \infty$	10^{-21} s.	All charged particles.
Strong Interaction	$\sim 10^{-13}$ cm.	10^{-23} s.	Hadrons.

A) Gravity:-

Gravity is by far the most familiar of these fields. The source of gravity is mass. Gravity is always manifested into the form of an attraction between two masses and is always directed along the line joining them. Gravity is independent of all other attributes of the bodies acted upon such as their electric charges, spins, directions of motion, etc....

The effects of gravity are observable only because it is of a long range. But it is weak when comapred with other fields, as seen from its characteristic time ($\sim 10^{16}$ s. or $\sim 10^{9}$ years) which is comparatively quite large. In the micro-world effects of gravity are usually negligible. Only in esoteric rigions, such as black holes, do the effects of gravity on elementary particles become important.

The gravitational field (interaction) is described classically by **Newton's famous inverse square law** i.e $F = Gm_1m_2/r^2$, where m_1, m_2 are the gravitational masses and "r" their distance apart. Another famous formula, which is also due to Newton, is F = ma, where "m" is the inertial mass and "a" being its acceleration. The equivalence between gravitational and inertial masses has led **Einstein** to speculate on the identification of the effects of gravity with those of acceleration. This is known as **the principle of equivalence** which forms one of the starting points of **General Relativity (G.R.)**.

G.R. is one of the theories which depend on **geometrical structure** to describe physics. It is the most successful theory in describing gravitational phenomena. It is important to remember that **G.R.** is still a classical theory, as all trials towards its **quantization** are, so far, **not satisfacotry.**

A successful quantum theory of gravity has not yet been formulated and the reconciliation of **G.R.** with quantum mechanics is one of the major outstanding problems in theoretical physics.

B) The Electromagnetic Field:-

The electromagnetic field (interaction) is strong (as compared with gravity and the weak interaction) and of a very long range. The source of this field is the electric charge which can be either positive or negative, and thus the interaction is an attraction when the two charges are different, and a repulsion when they are similar. At very small distances ($\sim 10^{-13}$ cm.) the electromagnetic field is