PROBLEMS IN CHEMICAL BOUNDARY LAYER

A THESIS

Submitted to the
Department of Mathematics
Faculty of Science
Ain Shams University

For The Degree of Ph. D.
in Applied Mathematics

By

EL-SAYED MOHAMED ABDEL RAHMAN EL-BASHBISHY

Under the Supervision of

Prof. Dr. M. A. KHIDR
Head of the Department of
Mathematics

Prof. Dr. A. G. EL-SAKKA
Professor of Applied
Mathematics

Faculty of Science
Ain Shams University

1987

الله

بسم الله الرحمن الرحيم

وقل اعملوا فسيرى الله عملكم ورسوله والمؤمنون

« صدق الله العظيم »

بسم الله الرحمن الرحيم

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Prof. Dr.M.A. Khidr for suggesting the problems solved in this thesis.

I am also very grateful to both my supervisors Prof. Dr. M.A.Khidr and Prof. Dr. A.G. El-Sakka for their continuous advices, encouragement and valuable discussions throughout the preparation of this thesis.

CONTENTS

Page

SUMMARY

SUMMARY

This thesis concerns with the investigation of some problems in boundary layer theory. The considered problems are related to the flow of hot gas which reacts chemically with a solid wall. We consider only the case of reactions of second order like the reaction of oxygen with a solid wall containing carbon.

In the first chapter we give a concise about the basic concepts in boundary layer theory, chemical kinetics and mass transfer.

Chapter two contains the investigation of the steady two-dimensional flow in which a second order reaction takes place between a semi-infinite solid horizontal plate and a surrounding gas assuming that:-

1. Schmidt's number is equal to 1.
2. The temperature at the plate is considered to be constant and high enough to cause the chemical reaction.

Also chapter two contains the investigation of the boundary layer problem near a vertical plate.

The basic equations in the two problems are partial non-linear differential equations. We used the similarity method to transform them to the ordinary type. These non-linear differential equations were solved approximately by using the collection method, where the solution is exact at the boundary and at some other points inside the region, and is a good approximation at the other points.

The analysis of the solution shows that:-

1. The concentration boundary layer thickness in the horizontal and vertical cases are smaller than the momentum boundary layer thickness.

2. The magnitude of the shear stress on the vertical plate is greater than the magnitude of the shear stress on the horizontal flat plate.

3. The momentum boundary layer thickness for the vertical plate is smaller than its value for the horizontal flat plate.

4. The concentration boundary layer thickness in the case of the vertical plate is smaller than its value for the horizontal plate.

5. The magnitude of the mass flux in case of the vertical plate is greater than its value for the horizontal plate.

Chapter three concerns with the investigation of the flow of a hot gas which chemically reacts with the walls of a wedge. A similar approximate solution is found for the problem in two cases. These cases are for the wedge with angles equal to $\frac{\pi}{2}$ and $\frac{\pi}{3}$.

The main results and conclusions of this chapter are:-

1. In all cases the concentration boundary layer thickness is always smaller than the momentum boundary layer thickness.

2. The magnitude of the shear stress increases when the angle of the wedge increases.

3. The magnitude of the mass flux decreases when the angle of the wedge increases.

4. The concentration boundary layer thickness decreases when the angle of the wedge increases.

By the same treatment we obtained an approximate analytical solution for the problem in which a gas flows between two non-parallel plates and when a second order chemical reaction takes place.

As in chapter two and chapter three, in chapter four, the concentration boundary layer thickness is found to be less than momentum boundary layer thickness in this case also.

All the obtained results are illustrated graphically.

CHAPTER ONE

CHAPTER ONE

INTRODUCTION

The aim of this thesis is to get the approximate solution of some problems in chemical boundary layer. These problems are related to the effect of chemical reactions of second order, when the temperature is considered to be constant and high enough to cause the reaction.

The problems of boundary layer with chemical reactions turn up investigations of various processes; for example: burning of fuel in hybrid engines , interaction of hypersoinc flying vehicles with atmosphere and the spreading of flame over the surface. For all the above processes it is necessary to calculate the mass transfer between a chemically reacting gas boundary layer and a reacting surface.

Boundary Layer Theory

The theory of laminar boundary layer was initiated by German Scientist Ludwing Prandtl in 1904. Prandtl,in a paper on "fluid motion with very small friction" made a hypothesis which states that for fluids with very small viscosity, the flow about a solid body can be divided into two regions:-

1. A very thin layer in the neighbourhood of the body (known as the boundary layer) in which the viscous effect may be considered to be confined.

2. The region outside this layer, where the viscous effects may be neglected and the fluid is regarded as inviscid.

With the aid of this hypothesis he simplified the Navier-Stokes equations to a mathematically tractable form which are then called the boundary layer equations.

The viscosity effect appears clearly in the thin layer very close to the body and that the remainder of the flow field could be considered a good approximation and treated as inviscid, that is, could be calculated by the methods of potential flow theory.

This thin region near the body surface is known as the boundary layer. A fundamental assumption of the boundary layer approximation is that the fluid immediately adjacent to the body surface is at rest relative to the body, an assumption that appears to be valid except for very low-pressure gases, when mean free path of the gas molecules is large relative to the body. Thus the hydrodynamics or momentum boundary layer may be defined as the region in which the fluid velocity changes from its free stream, or potential flow, value to zero at the body surface. If the boundary layer thickness is very small relative to all other flow dimensions, it is apparent that, for the two-dimensional boundary layer representation, the following conditions must prevail within the boundary layer [1] .

$$u \gg v$$

$$\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial y} \gg \frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial y} \gg \frac{\partial v}{\partial y}$$

where u and v are the components of velocity in directions ox and oy respectively.

When there is a mass transfer between the fluid and the surface, it is also found that in most practical applications the major concentration changes occur in a region very close to the surface . This gives rise to the concept of the concentration boundary layer, and again the relative thinness of these boundary layer permits the introduction of the boundary layer approximations similar