OPTIMIZATION PROBLEMS

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THESIS

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CONTENTS

SUMMARY

SUMMARY

In the present work, a study of vector optimization problems (VOP) is made. Several new combined problems are introduced for characterizing the efficient set of vector optimization problem. This thesis introduces new approach which combines the characteristics of both the K^{th} objective lagrangian problem and the proper equality constraint problem. Also, some of the basic notions in parametric convex programming are redefined and analyzed for the proposed approach. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind.

So we introduce new technique which give decision maker freedom in choosing the solution among the complete nondominated solutions using L_p-metric [generalized distance function]. And new algorithms for determining the best compromise set using goal programming via min-max techinque, the complete compromise set, the complete efficient set via goal programming technique, direct relation between multicriterion decision making approaches and goal programming technique.

This thesis contains three chapters:

Chapter I : Presents a survey on the most important theorems and methods for solving VOP problems.

Chapter II : Introduces the formulation and analysis of a combined forms for characterizing the efficient solutions of VOP.

Chapter III: Introduces several algorithms for treating multi-criterion decision making problems.

- An algorithm for determining the compromise set for linear programming problems using L_p-metric space (Alg I).
- An algorithm for determining the best compromise set using min-max technique via goal programming (Alg II).
- An algorithm for determining the complete efficient compromise set using goal programming (Alg III).
- An algorithm for determining the complete efficient set (Alg IV).
- An algorithm for determining a given solution to multi-criterion decision making problem using goal programming (Alg V).

Also:

The main results of the present work are included in [25], [26].

INTRODUCTION

Mathematical programming is an important branch in the field of operations research. It deals with the formulation and solution of optimization (minimization, maximization) problems. As a consequance the multicriterion decision making problem, or vector optimization problem (VOP) appears when decision maker must take a decision which optimize more than one conflicting objectives. It is a fundamental principle in VOP problems that the solution of these problems can be characterized as a solution of a certain single objective optimization problem, or a scalar optimization problem. The solution of VOP is referred in literature as efficient, non-inferior, pareto-optimal or nondominated solution (N).

The most common strategy is to characterize noninferior solutions in terms of optimal solutions of appropriate scalar optimization problems (SOP). Among the many possible ways of obtaining a scalar problem from VOP. There are common forms of scalarization to characterize the nondominated solutions of VOP which are the weighting problem $P(\omega)$, the $k^{\underline{th}}$ objective Lagrangian problem $P_k(u)$, the $k^{\underline{th}}$ objective ϵ-constraint problem $P_k(\epsilon)$, the generalized Tchebycheff norm $P(\beta)$ and the extended generalized Tchebycheff norm (P_β^α).

The ability of solving VOP relies completely on the success of solving the resulting SOP. Therefore a great deal of work has been done to characterize the efficient solution of VOP using the corresponding SOP.

Yu, (1974) introduced a generalized concept for efficiency which is based on the domination structure. The concept of non-inferior solution was introduced at the turn of the century by pareto, (1986).

Kuhn and Tuker, (1951) published a necessary and sufficient conditions for (proper) noninferiority.

Gass, Saaty and Zadeh solved VOP using the weighting problem. Haimes [5] introduces other forms of scalarization which are the Lagrangian problem and the constraint problem.

Lin [15] proposed another SOP to generate the efficient solutions of VOP which named by the proper equality constraint problem.

Bowman [3] showed that the solution of VOP can be characterized in terms of the generalized Tchebycheff norm problem and Choo. gave an extension to the generalized Tchebycheff norm to characterize the proper efficient solution of VOP even in the non convex case.

Wendell and Lee [5] combined the characteristics of both the weighting problem and the constraint problem to introduce another form of (SOP) which is called the Hybrid approach problem.

Elsawy, Mohamed and Osman & Dauer introduces several approaches which combined the characteristics of the weighting, Lagrangian, proper equality, k^{th} - ε constraint approach, the generalized Tchebycheff norm and the extended generalized Tchebycheff norm.

For these approaches the notions of the set of feasible parameters, the solvability set, the stability sets, were defined and analyzed for parametric convex nonlinear programming problems.

The importance of these results in VOP can be seen by the fundamental role which parametric techniques play in vector optimization problems [19, 20, 21].

On the other hand, goal programming is designed to solve the problem that satisfying (possible conflicting) goals as best as possible when some of the goals have a higher priority than other, which introduced by A. CHARNES and W. Cooper [13].

Y. IJIRI [13] used a generalized inverse approach to study this problem. The modification of the simplex method introduced by Charnes and Cooper for solving linear goal programming was throughly developed by S. Lee [13, 14] who also presented number of applications. All of these approaches are restricted to linear systems.

DAUER J. P., and KRUEGER [7] introduced an iterative algorithm for solving general goal programming problems. This approach enables one to solve linear, non linear, integer and other goal programming problems using the corresponding optimization technique in an iterative manner, this method allows a sensitivity analysis equivalent to that available for the corresponding optimization technique.

SCHNIEDERJANS and JWAK (1982) proposed new approach for solving general goal programming models, LEMKE (1954) originally presented the dual simplex algorithm for linear programming.

DAVID L. OLSON (1984) can pared Four algorithms in terms of accuracy and time requirements; these algorithms are Lee's modified goal programming simplex, revised simplex program developed by OLSON, Arthur and Ravindran's hierarchical solution procedure and Schnieder Jans and Kwak's dual simplex approach. He concluded that the dual simplex method appears to have superior computational times for models with a large proportion of positive deviational variable in the solution. The revised simplex algorithm appears more consistent in time and accuracy for general goal programming models.

FLAVELL (1976) introduces the notion of the best compromise set using goal programming approach via min-max technique [11].

CHAPTER I

MULTICRITERION DECISION MAKING, THEORY AND METHODOLOGY, A SURVEY