Ain Shams University
Faculty of Engineering

Buckling of Grid Shells Using Bi-Quadratic B-Spline Finite Element

A Thesis
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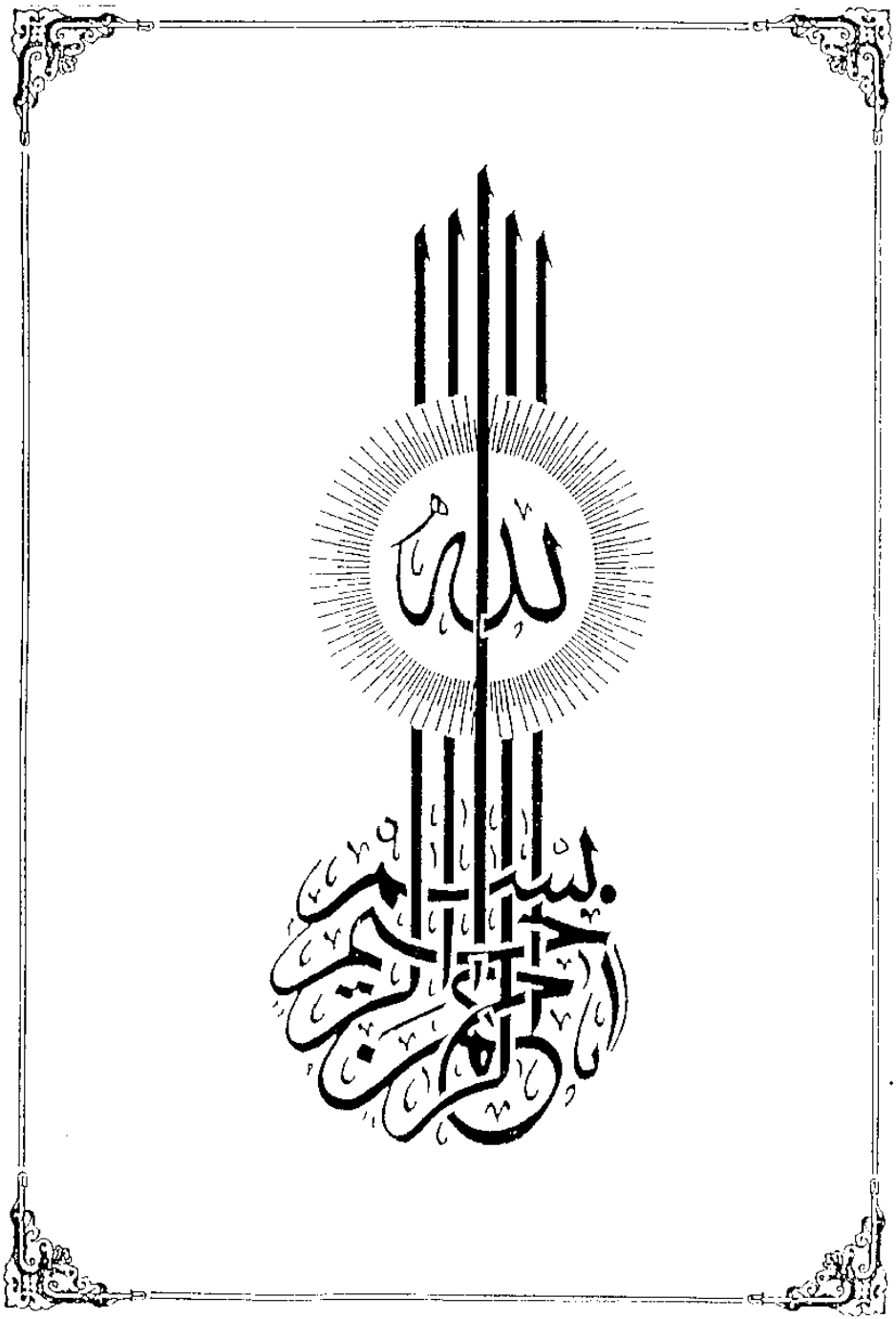

Statement

This dissertation is submitted to Ain Shams University for the degree of Doctor of Philosophy in Structural Engineering.

The research work reported in this thesis was carried out by the author in the department of Structural Engineering, Ain Shams University, from November 1992 to May 1997.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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Abstract

Name of Researcher :
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Title of Thesis :
"Buckling of Grid Shells Using
Bi-Quadratic B-Spline Finite Element"

This thesis aims to study the elastic stability of curved beams and grid shells which are represented geometrically by single or double free curves. The analysis of these types of structures is one of the more difficult problems that has been attempted with the finite element method. The notion of shells with free surfaces are characterized completely by the motion of a reference surface, and a finite element analysis is complicated greatly by the need to represent accurately the motion of this reference surface. Both the stretching and curvature change of the surface are required in the prediction of the strains existing throughout the shell.

This thesis presents a comprehensive study of the geometric representation of free curves and surfaces. A comparison between the different methods which are used to represent the curves and surfaces is made first, from which it can be concluded that the Quadratic B-spline polynomial is very efficient to represent the free surfaces since it requires less number of elements and control points as well as less computations.

The strain energy equations of plane curved structures are derived first based on the Quadratic B-Spline function with three control points. Four parameters have a significant effect on the derivation of the strain energy equations of plane curved structures. These parameters are :

(i) Change of curvature of the curved structure.
(ii) Axial strain of the curved structure.
(iii) Boundary conditions
(iv) External forces.

The stability of straight struts and compressed beams with initial imperfections as well as circular beams subjected to different types of loads are studied using Quadratic-B-Spline finite element. A comparison between the results and those of finite element method and Cubic-B-spline finite element method is made. After developing the strain energy equations of plane curved structures, the strain energy equations of grid shell is derived based on Bi-Quadratic B-Spline finite element with nine control points. To derive this equation, the effect of five parameters on the strain of grid shell has to be considered. These parameters are :

(i) Change of Normal Curvature.
(ii) Change of goedesic curvature.
(iii) Torsional bending term.
(iv) Boundary conditions
(v) External loads.

The stability of hyperbolic paraboloid grid shells and elliptic paraboloid grid shells as well as cylindircal grid vaults subjected to vertical loads are studied using quadratic B-Spline elements.

This element is capable of finding to a good degree of accuracy the failure load, the mode of buckling and load-deflection curve of grid shells.

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