

ON SCATTERING OF  $\mu^+$  MESONS

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In memory of

my father



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All gratitude is due to god almighty who guided and aided me to bring forth to light this theses.

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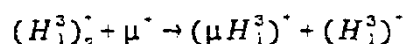
## SUMMARY

The present work has been based on the study of collisions of elementary particles.

An introduction gives a brief summary of the properties of tritium and  $\mu$ -mesons.

This thesis consists of four chapters.

The first chapter contains the mathematical derivation of the differential cross-section for the reaction,



using the distorted wave method.

The second chapter contains the calculations of the two wave functions corresponding to the relative motion of  $\mu^-$ -meson in field of  $(H_1^3)_2^+$  molecule and of  $(\mu H_1^3)^+$  in the field of  $(\mu H_1^3)^+$  molecule.

The difference between the corresponding levels of vibrational and rotational energies of  $(\mu H_1^3)^+$  and  $(H_1^3)_2^+$  molecules has been calculated.

The third chapter contains the calculations of the two wave functions representing the motion of the electron in the field of  $(H_1^3)_2^+$  and  $(\mu H_1^3)^+$  molecules and the integrations of the electronic wave functions over the element  $dr$  are calculated.

The fourth chapter contains the calculations for excited states of the two wave functions corresponding to vibrational and rotational motion of the nuclei of  $(H_1^3)_2^+$  and  $(\mu H_1^3)^+$  molecules. Also the integration over the elements  $p_r$ ,  $p'$  are calculated.

The differential cross-section for the present reaction are calculated for different values of energy levels.

It is found that calculated cross sections are comparable with the theoretical maximum ones.

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## *INTRODUCTION*



## INTRODUCTION

In this thesis we are going to study a chemical reaction, so we have to know first some properties of the components of this reaction. For the present section, these components are tritium and  $\mu$ -mesons.

### *Tritium*

The Tritium is the heaviest isotope of the element hydrogen and the only one which is radioactive. Tritium occurs in very small amounts in nature but is generally prepared artificially by processes known as nuclear transmutations. It is widely used as a tracer in chemical and biological research and is a component of the so-called thermo-nuclear of hydrogen bomb. It is commonly represented by symbol  $H^3$ , indicating that it has an atomic number of 1 and an atomic mass of 3, or by the special symbol T. For information about the other hydrogen isotopes properties.

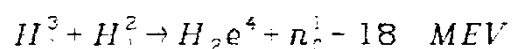
Both molecular tritium,  $T_2$ , and its counterpart hydrogen,  $H_2$ , are gases under ordinary conditions. Because of the great difference in mass, many of the properties of tritium differ substantially from those of ordinary hydrogen.

chemically, tritium behaves quite similarly to hydrogen. However, because of its larger mass, many of its reactions take place more slowly than do those of hydrogen.

The nucleus of the tritium atom, often called a triton and symbolized  $t$ , consists of a proton and two neutrons. It has a mass of 3.01700 atomic mass units (amu), a nuclear spin of  $\frac{1}{2}$ , and a magnetic moment of 2.9788 nuclear magnetons.

It undergoes radioactive decay by emission of a  $\beta$ -particle to leave a helium nucleus of mass 3. No  $\gamma$ -rays are emitted in this process. The half-life for the decay is 12.26 years. The most energetic of the  $\beta$ -particles emitted by tritium have the comparatively low energy of 18.6 Kilo electron volts (KeV),  $\beta$ -particles are completely stopped by 7 mm of air or by 0.01 mm of paper or similar material. The average energy of the  $\beta$ -particle is 5.69 KeV.

When tritium is bombarded with deuterons of sufficient energy, a nuclear reaction known as fusion occurs and energy considerably greater than that of the bombarding particle is released. The reaction may be written as

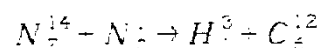


one of those which supply the energy of the thermo-nuclear bomb. It is also of major importance

in the development of controlled thermo-nuclear reactors. Enormous quantities of tritium will be required if such reactors are perfected and brought into use as electric power generators.

### *Natural Occurrence*

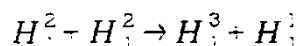
Before the start of thermo-nuclear weapons testing in 1954, rainwater contained approximately 1-10 atoms of tritium per  $10^{18}$  atoms of hydrogen. Such tritium originator largely from the bombardment of nitrogen in the upper atmosphere with neutrons and protons from cosmic rays, as in reaction (2), because the half-life



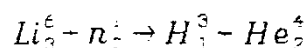
of tritium is short in comparison with the time required for mixing of the ocean waters, the concentration of tritium in the ocean is much lower than in rainwater—before 1954 the total amount of tritium on the Earth's surface was estimated at 1800 g. Of which about 11g was in the atmosphere and 13g in groundwaters. Testing of thermo-nuclear weapons has resulted in sharp rises in the tritium content of rainwater to values as high as 500 atoms per  $10^{18}$  atoms of hydrogen.

## Preparation

Tritium was first produced in the laboratory by bombarding compounds of deuterium with high-energy deuterons, as in the reaction



A number of other nuclear reactions also give rise to tritium. The most important of these is the absorption of slow neutrons by the lithium isotope of mass 6. According to reaction . By irradiating



enriched lithium-6, in the form of an alloy with magnesium or aluminum, with neutrons from a nuclear reactor, tritium may be prepared on a large scale. Uses As a result of its production for use in nuclear weapons, tritium has become available in large quantities at very low cost. It is used in admixture with zinc sulfide in the production of luminous paints, which have largely replaced the radium formerly used on watch dials, such mixtures are also used to produce small, permanent light sources.

Tritium absorbed on metals is used in targets for the production of fast neutrons by bombardment with deuterons. Tritium has been much used in hydrological studies, since it is an ideal tracer for water movement.

Some studies depend on natural tritium or that introduced by weapons testing in other cases large amounts of tritium are deliberately added.

Investigations include the distribution of groundwater in oil fields; the tracing of springs, rivers, and lakes; water seepage and loss from reservoirs; and the movement of glaciers.

Tritium has also been used as a tracer for hydrogen in the study of chemical reactions. The most widespread use of tritium has probably been in biological research, where it has been used both as a hydrogen tracer and as a molecular label in studies of metabolism, biosynthesis, and cytology. In particular, tritiated thymidine and other nucleotides and nucleosides have been extensively used in studies of the formation of DNA and RNA.

Compounds. very few compounds of pure tritium have been prepared and studied. Such compounds would undergo decomposition quite rapidly under the action of the tritium  $\beta$ -radiation. Tritium oxide,  $T_2O$ , has been prepared by oxidation of tritium gas with hot copper oxide or by passing an electric spark through a mixture of tritium and oxygen. Its melting point is  $4.49^\circ C$ , compared with  $0^\circ C$  for ordinary water. Of much greater importance are compounds, especially organic compounds, in which a small fraction of the hydrogen atoms have been replaced by tritium. Such labelled compounds

are employed in tracer studies, such as those indicated above. Tritium-labelled compounds may be prepared by ordinary synthetic chemical methods, such as the catalytic addition of tritium-hydrogen mixtures to unsaturated compounds. Tritium may be exchanged for hydrogen in the presence of a catalyst such as platinum or a strong acid.

In recoil labelling, a mixture of an organic compound and a lithium salt are irradiated with neutrons in a nuclear reactor; some of the energetic tritons produced are incorporated into the organic compound.

Another important labelling procedure consists of the exposure of an organic compound to tritium gas in a sealed vessel; the tritium  $\beta$ -radiation facilitates the exchange of hydrogen in the compound with tritium in the gas. Some compounds of biological interest have been prepared by growing organisms in tritiated water.

### *Analysis*

Because of its weak  $\beta$ -radiation, tritium is not readily measured by the ordinary Geiger-Müller counter. More efficacious is the introduction of tritium as a gas inside the counting tube. Alternatively, the ionization of a gas caused by the  $\beta$ -radiation may be measured in an ionization

chamber, or the tritium compound may be dissolved in a suitable solvent containing a phosphor and the light pulses excited by the  $\beta$ -particles then may be counted with a scintillation counter. Tritium gas containing only small amounts of ordinary hydrogen may be analyzed with a mass spectrometer or by measuring the density of the gas. Because of the very short range of the tritium  $\beta$ -particle, auto-radio-graphy, the exposure of radioactive material to a photographic plate is often used to locate precisely the position tritium in biological material. [13]

### *The $\mu$ -meson*

Studies of cosmic rays began after the first world war and it has been one of the most fruitful in modern physics leading to the discovery of the positron, the  $\pi^+$  and  $\pi^-$  mesons and the strange particles. The importance of cosmic rays in geophysical and cosmological research generally is also very great.

A search was made for particles of this order of mass in the radiation at sea level and Anderson, followed shortly after by Street and Stevenson, did discover them (include chamber photographs produced by the hard component, because their masses are intermediate between those of electrons and of