Study of Some Properties of The Product of Infinite Series

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BY

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بسم الله الرحمن الرحيم

سبحانك

لا علم لنا إلا ما علمتنا إنك أنت العليم الحكيم

صدق الله العظيم

بسم الله الرحمن الرحيم

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THE STUDENT HAS STUDIED THE FOLLOWING COURSES AND PASSED THEM SUCCESSFULLY IN PARTIAL FULLFILMENT FOR THE M.Sc. DEGREE /

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SUMMARY

This thesis deals with some methods for summing infinit divergent series, such as the method given by E. Cesàro who was able to define the idea of convergent method (C, 1) and its generalisation (C, K), The method of Holder who defined the convergent method (H, K) and the method of N.H. Abel who is considered to be the third contributer in this field. Abel's theorem was applied in the beginning of the 19^{th} century.

The theory of summability has many uses throughout mathematics. The mathematicians should have a basic understanding of and working ability in the area of summability. The engineer or physicist who deals with Fourier series, Fouries transforms, or analytic continuation will find the concepts of summability theory invaluable to his research.

The thesis dealt with the properties of the summability methods and the relations which link each other. We studied the properties of multiplication of series in a new way, that is, circuler method, curves method and step function method. Before, there were two methods, the Cauchy method and the square method.

This thesis consists of four main chapters. The first chapter contains some basic notation and definitions of the summability series. We displayed different summability method such as (C, K) method (Cesàro method) and its properties, (E, K)method Holder method (E, K) and its properties. We discussed equivalence between both theories Holder's and Cesàro when the rank (K) is a round positive figure, Abel's method and its properties, and its relation with Cesàro. Finally we defined the general summability method and its properties.

In the second chapter we studied Ccuchy method for multiplication of infinit series and the properties of its result. This method shows that the result of the product of two infinite convergent series according to Cauchy is not necessarily convergent. We discussed the theory of Merten and proved that its conditions are necessary. Then we debated , the summable multiplication series (c, 1) cesaro, by convergent series.

We concluded a new method for the multiplication of two series. One of them is absolutly convergent while the other is convergent. Which the circuler method. It is proved that the result of multiplication is convergent. We concluded, as well, another method that is δ_n Curves for the multiplication of two series one is absolutly convergent while the other is conditionaly convergent.

The third chapter delas with summability convergent factors and their properties from the first and second kind. The properties of summability convergent factor by Cesàro method (C,α) are studied. These properties are very important for following chapter.

In the fourth chapter we discussed the properties of multiplication of convergent series or divergent series and the properties of the result of the multiplication by Cauchy of two series one of them is summable by Cesàro (C, K) and the other is summable (C, S). The result can be summed by Cesàro (C, K+S+1) method. We studied the result of the product of two series one of them is convergent and other is (C, K) summable by square method. We concluded a new method for the multiplication of two series one of them is absolutely convergent and while the other is summable by Cesarc method. This method is called stepfunction method. It is proved that the result of the multiplication by this method is summable by Cesàro

(C, 1). We treated the same problem when one of the series is absolutly convergent and the other is summable by Cesàro (C, K). It is proved that the result of the multiplication is summable Cesàro (C, K) method under certain conditions.

CHAPTER 1

Introduction of summability method

§1.1. Introduction:

The most important idea in analysis is that of convergence. Summability methods serve generalization of the classical notation of convergence. Our problem is to associate a generalized limit to a convergent or divergent sequence or Series.

The principle adopted in the summability method is, if the series $\sum_{n=0}^{\infty} a_n$ is given, we apply the Transformation (T) to the sequence of partial sums A_n ($A_n = \sum_{\nu=0}^{n} a_\nu$) of the series, denoted by $t_n = T(A_n)$, or the transformation may be obtained as a function of a continuous variable ($t(x) = T(A_n)$). Then we study the ordinary convergence of the newly obtained sequence or function. We say that the given series is summable by the method T if the transformed sequence or function tends to a limit.

Definition : [6], [13]

We say that $\lim_{n\to\infty} A_n = L$ (T) if and only if $t_n \longrightarrow L$ as $n \longrightarrow \infty$ (or $t(x) \longrightarrow L$ as $x \longrightarrow 1^-$) we give here only four summability methods, though other summability method are there.

§ 1.2 Cesàro method (C, K), [6].

1.2.1. Statement of the method: [6]

Let $\sum_{n=0}^{\infty} a_n$ be a series of real (complex) numbers,

and

$$\left.\begin{aligned} A_n^0 &= A_n = a_0 + a_1 + \dots + a_n , \\ A_n^1 &= A_0^0 + A_1^0 + A_2^0 + \dots + A_n^0 , \\ A_n^2 &= A_0^1 + A_1^1 + A_2^1 + \dots + A_n^1 , \\ &\vdots \\ A_n^k &= A_0^{k-1} + A_1^{k-1} + \dots + A_n^{k-1} , \end{aligned}\right\} \qquad (1.1)$$

In the special case where $A_n = 1 \; \forall n \in I$ we denote A_n^k by E_n^k. Now let.

$$\sigma_n^k = \frac{A_n^k}{E_n^n}$$

If $\lim_{n\to\infty} \sigma_n^k = A$, then (1.2)

we say that $\sum_{\nu=0}^{\infty} a_\nu$ is summable (C, K) to A, and we write.

$$\sum_{\nu=0}^{\infty} a_\nu = A \; (C, K)$$

In order to express A_n^k in terms of A_n we mention the following formal relation: