

AIN SHAMS UNIVERSITY  
FACULTY OF ENGINEERING

NON-LINEAR ELASTIC-PLASTIC ROTATION CAPACITY OF  
NON-RECTANGULAR AND PITCHED-ROOF STEEL FRAME ELEMENTS

BY  
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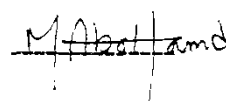
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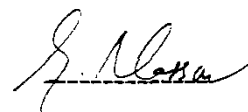
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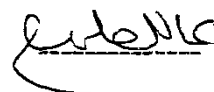
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## STATEMENT

This dissertation is submitted to Ain Shams University for the degree of M. Sc. in Civil Engineering .

The work included in this thesis was carried out by the author in the Department of Civil Engineering , Ain Shams University , from Nov. 1990 to Jan . 1995 .

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution .

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**SUMMARY OF MASTER  
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BY  
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**NON-LINEAR ELASTIC-PLASTIC ROTATION CAPACITY OF NON-  
RECTANGULAR AND PITCHED-ROOF STEEL FRAME ELEMENTS**

The object of this research is to investigate the elastic-plastic non-linear behaviour of non-rectangular and pitched-roof steel frame elements . The thesis contains seven chapters :

**Chapter one:**

Deals with the introduction and scope of the the thesis .

**Chapter two:**

Deals with the historical review and the literature survey on elastic-plastic behaviour and rotation capacity of frames under different types of loading.

**Chapter three:**

Includes the finite deflection analysis theory used in the present work and the steps leading to the formulation of a computer program for dealing with the problem . The program is used to study the rotation capacity of pitched roof frames.

**Chapter four:**

A parametric study is carried out to investigate the effect of change of frame geometry and dimensions of frame joints under two types of loading patterns on the nonlinear rotation capacity before failure .

**Chapter five:**

Includes a study of the effect of local buckling of the cross section frames on the rotation capacity of frames. A parametric study is carried out to show the effect of increasing the moment of inertia of most stressed parts of members in reducing the local buckling effect of the flange plates .

**Chapter six:**

A study on the effect of increase of number of bays on the rotation capacity of frames is presented.

**Chapter seven:**

Includes summary , conclusions and recommendations for future extension of the present research.

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## CHAPTER (1)

### INTRODUCTION

#### 1-1 INTRODUCTION

Actual behaviour of steel frames is very different from the way assumed by many designers. Elastic design in many cases may hide the real behaviour which is dangerous. The actual factor of safety of a structure can be reduced than the designer thinks. In other cases the real factor of safety may be more, and an uneconomic structure will result.

Long ago, research started to know the behaviour of structures mainly based on the plastic theory and later plastic design was allowed in many countries.

The quick advancement of plastic design is mainly a result of a better structural economy through a well distribution of the used material. This is obtained by taking facility of the reserve of strength of structural steel besides the elastic limit, by using correct methods of analysis, and by confirming a uniform factor of safety against failure for all structures.

Plastic analysis of steel structures depends on capacity of members to form plastic hinges and to redistribute moments. In order for redistribution of moment to take place, certain plastic hinges must sustain their plastic moment through some angle of rotation

The amount of rotation necessary may affect the stability of the structure and therefore , may affect the geometry of the structural shapes .

The ability of a structure to rotate near its collapse mechanism is defined as the " rotation capacity " . The rotation capacity can be thought of as the warning time the structure has between the formation of the first plastic hinge and the structure collapse by mechanism . The increase of this value is a good notification that sudden collapse of this structure is not expected .

The objectives of the research presented in this thesis are :-

- 1- Investigating the non-linear elastic-plastic behaviour of steel pitched - roof frames under different factors which affect the rotation capacity of members such as :-
  - a) Geometry and frame dimensions .
  - b) Load acting on frame , and
  - c) Overall buckling of frame .
- 2- Studying the effect of local buckling of members on the ability of the structure to form plastic hinges .
- 3- Carrying out a parametric study to improve the rotation capacity against the effect of local buckling .

## 1-2 SCOPE

The present thesis contains seven chapters and one appendix :-

Chapter one deals with the introduction and scope of the thesis .

Chapter two deals with historical review and literature survey on the elastic-plastic behaviour and rotation capacity of frames under different types of loading .

Chapter three , where the finite deflection analysis theory , used in the present work , is detailed and the steps leading to the formulation of a computer program are explained . The program is used to study the rotation capacity requirements for nonlinear elastic - plastic behaviour of pitched - roof frames.

Chapter four , where a parametric study is carried out to investigate the effect of geometry and dimensions of frames. (under two types of loading patterns ) , on the rotation capacity by using cross section S. I .B. and steel No. 44 having  $f_y = 2600 \text{ Kg/Cm}^2$

Chapter five , an attempt is made to study the effect of local buckling on the rotation capacity of frames . A parametric study is carried out to show the effect of increasing the moment of inertia of parts of members in reducing the local buckling effect .

Chapter six , an introduction to study the effect of number of bays on the rotation capacity of frames is presented .

Chapter seven , includes a summary conclusions and recommendations for future extension of the research .

## CHAPTER (2)

### HISTORICAL REVIEW

#### 2-1 INTRODUCTION

The plastic analysis is now widely introduced in the design of steel structures . Many researchers have tried to study the behaviour of structures beyond the elastic limit to determine the theoretical failure loads of steel **frames**. Plastic analysis of steel structures is based on the capacity of members to form plastic hinges , and to redistribute moments . So, a big work has been done by researchers to study the factors affecting the rotation capacity and investigate the elastic-plastic behaviour of structures under different types of loading.

In this chapter a historical reconsideration of the previous research work in the field of plastic analysis and rotation capacity requirements for structural frames is forwarded.

#### 2-2 THEORETICAL FAILURE LOADS

The elastic-plastic analysis is considered to be the most general and precise system for predicting the theoretical ultimate strength of a frame .

In this method, the load-displacement relationship is evaluated under the action of increasing loads . When a plastic hinge is formed , its effect , is taken into account in the following calculations . The ultimate carrying capacity is reached when the combination of progressive yielding , axial force and joint displacements reduces the stiffness of the