AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

TWO-DEGREE-OF-FREEDOM GYRO DYNAMICS
AND THE EFFECT OF ITS ERRORS ON THE PERFORMANCE OF A STRAPDOWN INERTIAL NAVIGATION SYSTEM

BY ENGINEER

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A THESIS

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Statement

This dissertation is submitted to Ain Shams University for the degree of Master of ·cience in Mechanical Engineering.

The work included in this thesis was carried out by the author in the Department of Mechanical Engineering, Ain Shams University, from Dec. 1991 to Dec. 1994.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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Synopsis

The sources of errors originating from the Two-Degree-of-Freedom yroscopic sensor in the ballistic missile navigation systems can represent an nportant item in the missile accuracy. Certainly sure, the different components in he navigation, autopilot and guidance loops greatly affect this accuracy, but any rial directed to improve these systems effectiveness without deep analysis of the gyroscopic sensor usually leads to vane.

The gyro element in the missile navigation system has an extremely complex dynamics, and the publications existing in this concern are not sufficient to provide the designers by the required data.

For the previous reasons, this thesis is devoted to a detailed study of the Two-Degree-of-Freedom gyro sensor of the missile navigation system and autopilot.

The thesis include mainly:

1- Nonlinear and linearized modeling of the mechanical Two-Degree-of-Freedom gyro, and solving these models for the important input cases.

2- Determination of the performance equations of the mechanical Two-Degree-of-Freedom gyro and evaluation the different sources of errors.

3- Studying and exploring the error equations of the strapdown inertial navigation systems and study the effect of the error sources of the mechanical Two-Degree-of-Freedom gyro on the missile navigation system.

Having a detailed analysis concerning the previously given point, the navigation system designers can be satisfied and can be felt sufficiency of information about that important system.

Contents

List of Figures And Tables

List of Symbols

$,\underline{b},\underline{d}$	Vectors
$,c$	Dimensions of rotor
^{b}	Drift rate due to gimbal bearing friction
$\underline{l}^{p}$	Vector form of D^{p}
$\underline{l}^{u}$	Unbalance drift rate
l_x, d_y, d_z	Components of D^{p} in directions x, y, z
$\underline{f}^{a}$	Measured specific force
$\underline{f}^{b}$	Measured specific force in platform frame
$\underline{f}^{k}$	Computation frame specific force
$\underline{f}^{n}$	Acceleration vector in navigational frame
$(u)\underline{f}^{a}$	Uncertainty in $\underline{f}^{a}$
$\underline{g}^{k}$	Vector form of Γ^{k}
$\underline{g}'$	Acceleration vector
h	Altitude
δh	Altitude error
i	Current of torque generator
k	All scale factor errors
k^{a}	Scale factor asymmetry error
k^{b}	Bearing friction scale factor error
k^{c}	Basic scale factor error
k^{l}, k^{n}	Linearity errors
l	Longitude angle
δl	Longitude error
m	Unbalance mass, Mass of earth
n	Total spin
$\delta \underline{n}$	Navigation error vector
p	Differential operator
q_1, q_2	Forcing functions
$\underline{r}$	Offset vector, Position vector
$\delta \underline{r}$	Change in Offset vector, Error in position vector
$\underline{r}^{i}$	Inertial frame position vector
$\underline{r}^{k}$	Computation frame position vector
r_e	Earth radius

Geocentric position vector

$,r_2,r_3$ Components of $\underline{\mathbf{r}}$ in the **g** frame

Laplace operator

Output volt

Weighting factor

State variable vector

$,y,z$ Inertial frame

$_r,y_r,z_r$ Orthogonal case frame

$_t,y_t,z_t$ Torque generator frame

$_1,y_1,z_1$ Outer gimbal frame

$_2,y_2,z_2$ Inner gimbal frame

$_3,y_3,z_3$ Rotor frame

$\mathbf{A,B,D}$ Skew symmetric forms of $\underline{\mathbf{a}},\underline{\mathbf{b}},\underline{\mathbf{d}}$

$\mathbf{C}$ Damping factor

$\mathbf{C}_j$ Coefficients of equations ; $j = 1,2,3,4$

$\mathbf{C}_g^r$ Transformation matrix from frame **g** to frame **r**

$\underline{\mathbf{D}}$ Error drift coefficient vector

$\underline{\mathbf{D}}^a$ Anisoelasticity drift vector

$\mathbf{D}^b$ Gimbal bearing friction drift

$\mathbf{D}^h$ Motor hunting drift

$\mathbf{D}^p$ Skew symmetric matrix of error values

$\mathbf{D}^s$ Mean drift

$\mathbf{D}^t$ Torque generator drift coefficient

$\underline{\mathbf{D}}^u$ Unbalance drift vector

$\underline{\mathbf{D}'}$ Gyro drift vector

$\mathbf{D}_O$ Deviation of the normal

$\mathbf{D}_{1x},\mathbf{D}_{1y},\mathbf{D}_{1z}$ Outer gimbal viscous damping factors

$\mathbf{D}_{2x},\mathbf{D}_{2y},\mathbf{D}_{2z}$ Inner gimbal viscous damping factors

$\delta\mathbf{D}^t$ Error in torque generator drift coefficient

$\mathbf{E}^n$ Skew symmetric form of $\underline{\varepsilon}^n$

$\mathbf{F}^b$ Force acting on gimbal bearing

$\mathbf{F}^n$ Skew symmetric form of $\underline{\mathbf{f}}^n$

$, F_2$	Input functions
^{1}	Gravitational field vector in skew symmetric form
$_j(s)$	Open loop transfer functions ; $j = 1,2,3,4$
$\underline{H}$	Angular momentum vector
$\underline{H}^k$	Nominal angular momentum vector
$\underline{H}_f$	Angular momentum vector with respect to frame **f**
$H_j(s)$	Feedback transfer functions ; $j = 1,2,3,4$
H_r	Angular momentum of rotor
$\underline{H}_2$	Angular momentum vector of inner gimbal and rotor system
$\underline{H}_3$	Angular momentum vector of rotor
H_{3z}	Component of $\underline{H}_3$ in direction of z_3
H_{2x}, H_{2y}, H_{2z}	Components of $\underline{H}_2$ in directions x, y, z
$\delta\underline{H}$	Error vector in angular momentum
I	Identity matrix
$\hat{I}$	Operator to convert vectors to skew symmetric form
I_m	Imaginary part
I_2	Moment of inertia of inner gimbal and rotor system
I_{2x}, I_{2y}, I_{2z}	Components of I_2 in directions x, y, z
I_{3z}	Moment of inertia of the rotor in direction of z_3
J	Moment of inertia
K	Compliance tensor
K_{ij}	Components of K ; $i, j = 1,2,3$
K_e, K_p	Equatorial and polar constants
K_1, K_2	Gains
K_{1x}, K_{1y}, K_{1z}	Outer gimbal spring rates
K_{2x}, K_{2y}, K_{2z}	Inner gimbal spring rates
L	Latitude angle
L_c	Geocentric latitude
L_o	Initial Geographic latitude
δL	Latitude error
$\underline{M}$	Torque vector
$\underline{M}^b$	Gimbal bearing friction drift torque vector
$\underline{M}^c$	Command torque vector
$\underline{M}^d$	Disturbance torque vector
$\underline{M}^t$	Torque generator torque vector

$\underline{M}_r$ Unwanted torques vector

$\underline{M}_t$ Control torques vector

$\underline{M}_m$ Error torques and applied control torques vector

$M_j(s)$ Transfer functions ; $j = 1,2,3,4$

$\underline{M}_1$ External applied torque vector on outer gimbal

$\underline{M}_2$ External applied torque vector on inner gimbal

M_{1x}, M_{1y}, M_{1z} Components of $\underline{M}_1$ in directions x, y, z

M_{2x}, M_{2y}, M_{2z} Components of $\underline{M}_2$ in directions x, y, z

M_{1m}, M_{2m} Components of $\underline{M}_m$ on outer and inner gimbals

M_o^t Outer gimbal torque generator torque

$(u)\underline{M}^g$ Uncertainty torques vector

N Universal gravitational constant

N^n Skew symmetric form of v^n

P^k Transformation error angles matrix

Q_j, Q_1, Q_2 Forcing functions

R Resistance, Skew symmetric form of vector $\underline{r}$

R^k Nominal resistance

R_e Real part

S Actual gyro scale factor

δS Scale factor error

S^k Nominal gyro scale factor

S^t Torquer scale factor

S^T Command rate scale factor

S^{tk} Nominal torquer scale factor

δS^t Error in torquer scale factor

T^p, T^g Gyro torque scale factor uncertainty matrices

T_d Driving torque of the rotor

T_w Windage torque

T_1, T_2 Time constants

V^n Earth referenced velocity vector

V_N, V_E, V_D Components of V^n in navigational frame

δV Velocity error

X_i Input amplitude

X_O Output amplitude

X, Y Coefficients of equations

Z^j Initial system misalignment error matrix

Greek Litters

Input angle, error angle

$\alpha_1, \alpha_2, \alpha_3$ Misalignment angles

Misalignment matrix, bias

$_{ij}$ Misalignment angles ; $i, j = 1, 2, 3$

Uncertainty term

$\underline{\varepsilon}^n$ Attitude error vector

$\varepsilon^N, \varepsilon^E, \varepsilon^D$ Components of ε^n in navigation frame

ϕ_x, ϕ_y, ϕ_z Euler angles

ϕ_1, ϕ_2, ϕ_3 Misalignment angles

ϕ_O Amplitude of Euler angles

γ Angular displacement of rotor w.r.t magnetic field

γ_{ij} Misalignment angles term ; $i, j = 1, 2, 3$

θ_1, θ_2 Output angles of the gyro

θ_3 Rotor angular displacement in rotor frame

μ Friction coefficient, Product of earth mass with ,universal Gravitational constant

$\underline{\nu}^n$ Error angles vector

ν_N, ν_E, ν_D Components of $\underline{\nu}^n$ in navigational frame

$\underline{\rho}^k$ Vector form of P^k

ρ_x, ρ_y, ρ_z Components of P^k in directions x, y, z

τ_x, τ_y, τ_z Components of T^p in directions x, y, z

$\underline{\omega}$ Angular velocity

$\underline{\omega}^c$ Command rate

$\underline{\omega}^k$ Measured Angular velocity

$\underline{\omega}_f$ Angular velocity with respect to frame **f**

ω_n Natural frequency

$\underline{\omega}_p$ Angular velocity with respect to frame **p**

ω_s Schuler frequency

$\underline{\omega}_1$ Angular velocity of the outer gimbal

$\underline{\omega}_2$ Angular velocity of the inner gimbal

$\underline{\omega}_3$ Angular velocity of the rotor

$\underline{\omega}_{ie}$ Earth Angular velocity

$\omega_x, \omega_y, \omega_z$ Components of $\underline{\omega}$ in directions x, y, z

$_{px}, \omega_{py}, \omega_{pz}$ Components of $\underline{\omega}_p$ in directions **x, y, z**

$_{1x}, \omega_{1y}, \omega_{1z}$ Components of $\underline{\omega}_1$ in directions **x, y, z**

$_{2x}, \omega_{2y}, \omega_{2z}$ Components of $\underline{\omega}_2$ in directions **x, y, z**

$_{3x}, \omega_{3y}, \omega_{3z}$ Components of $\underline{\omega}_3$ in directions **x, y, z**

$\underline{\omega}_{ij}^{k}$ Angular velocity of frame **i** w.r.t frame **j** projected in frame **k**

$\underline{\omega}_{ip}^{g}$ Angular velocity of frame **i** w.r.t frame **p** projected in frame **g**

$\underline{\omega}_{ip}^{p}$ Angular velocity of frame **i** w.r.t frame **p** projected in frame **p**

$\underline{\omega}_{kj}^{j}$ Angular velocity of frame **j** w.r.t frame **k** projected in frame **j**

$\delta\underline{\omega}$ Error in $\underline{\omega}$

$(u)\underline{\omega}^{p}$ Gyro uncertainty in platform inertial angular velocity

$(u)\omega_x, (u)\omega_y, (u)\omega_z$ Components of $(u)\underline{\omega}^{p}$ in directions **x, y, z**

$\underline{\psi}^{n}$ Transformation error angles vector

ζ Damping ratio

$\zeta_x, \zeta_y, \zeta_z$ Components of Z^{j} in directions **x, y, z**

Γ^{k} Transformation error matrix

Λ Characteristic matrix

Ω_{ip}^{p} Skew symmetric form of $\underline{\omega}_{ip}^{p}$

Ω_{kj}^{j} Skew symmetric form of $\underline{\omega}_{kj}^{j}$

Ψ^{n} Skew symmetric form of $\underline{\psi}^{n}$

$(\tilde{\ })$ A measured value

$(\hat{\ })$ A calculated value

$(\dot{\ })$ Derivative w.r.t time