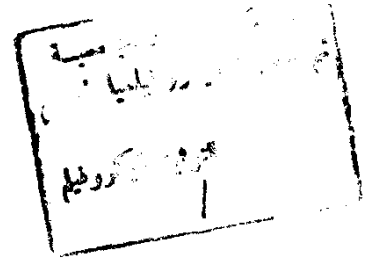


AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING



**TWO-DEGREE-OF-FREEDOM GYRO DYNAMICS
AND THE EFFECT OF ITS ERRORS ON THE PERFORMANCE OF A
STRAPDOWN INERTIAL NAVIGATION SYSTEM**

BY ENGINEER

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55363

A THESIS

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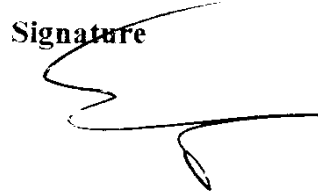
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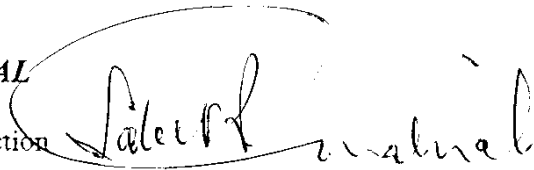
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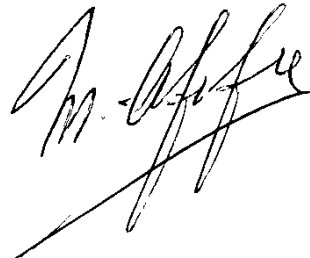
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Statement

This dissertation is submitted to Ain Shams University for the degree of Master of science in Mechanical Engineering.

The work included in this thesis was carried out by the author in the Department of Mechanical Engineering, Ain Shams University, from Dec. 1991 to Dec. 1994.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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Synopsis

The sources of errors originating from the Two-Degree-of-Freedom gyroscopic sensor in the ballistic missile navigation systems can represent an important item in the missile accuracy. Certainly sure, the different components in the navigation, autopilot and guidance loops greatly affect this accuracy, but any trial directed to improve these systems effectiveness without deep analysis of the gyroscopic sensor usually leads to vane.

The gyro element in the missile navigation system has an extremely complex dynamics, and the publications existing in this concern are not sufficient to provide the designers by the required data.

For the previous reasons, this thesis is devoted to a detailed study of the Two-Degree-of-Freedom gyro sensor of the missile navigation system and autopilot.

The thesis include mainly:

- 1- Nonlinear and linearized modeling of the mechanical Two-Degree-of-Freedom gyro, and solving these models for the important input cases.
- 2- Determination of the performance equations of the mechanical Two-Degree-of-Freedom gyro and evaluation the different sources of errors.
- 3- Studying and exploring the error equations of the strapdown inertial navigation systems and study the effect of the error sources of the mechanical Two-Degree-of-Freedom gyro on the missile navigation system.

Having a detailed analysis concerning the previously given point, the navigation system designers can be satisfied and can be felt sufficiency of information about that important system.

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List of Symbols

$\underline{b}, \underline{d}$	Vectors
\underline{c}	Dimensions of rotor
\underline{f}^b	Drift rate due to gimbal bearing friction
\underline{I}^p	Vector form of D^p
\underline{I}^u	Unbalance drift rate
$\underline{I}_x, \underline{d}_y, \underline{d}_z$	Components of D^p in directions x, y, z
\underline{f}^a	Measured specific force
\underline{f}^b	Measured specific force in platform frame
\underline{f}^k	Computation frame specific force
\underline{f}^n	Acceleration vector in navigational frame
$(\underline{u})\underline{f}^a$	Uncertainty in \underline{f}^a
\underline{g}^k	Vector form of Γ^k
\underline{g}'	Acceleration vector
h	Altitude
δh	Altitude error
i	Current of torque generator
k	All scale factor errors
k^a	Scale factor asymmetry error
k^b	Bearing friction scale factor error
k^c	Basic scale factor error
k^l, k^n	Linearity errors
l	Longitude angle
δl	Longitude error
m	Unbalance mass, Mass of earth
n	Total spin
$\delta \underline{n}$	Navigation error vector
p	Differential operator
q_1, q_2	Forcing functions
\underline{r}	Offset vector, Position vector
$\delta \underline{r}$	Change in Offset vector, Error in position vector
\underline{r}^i	Inertial frame position vector
\underline{r}^k	Computation frame position vector
r_e	Earth radius

- Geocentric position vector
- r_2, r_3 Components of \underline{r} in the \underline{g} frame
- Laplace operator
- Output volt
- Weighting factor
- State variable vector
- y, z Inertial frame
- y_r, z_r Orthogonal case frame
- y_t, z_t Torque generator frame
- y_1, z_1 Outer gimbal frame
- y_2, z_2 Inner gimbal frame
- y_3, z_3 Rotor frame
- A, B, D** Skew symmetric forms of $\underline{a}, \underline{b}, \underline{d}$
- C** Damping factor
- C_j Coefficients of equations ; $j = 1, 2, 3, 4$
- C_g^r Transformation matrix from frame \underline{g} to frame \underline{r}
- D** Error drift coefficient vector
- \underline{D}^a Anisoelasticity drift vector
- \underline{D}^b Gimbal bearing friction drift
- \underline{D}^h Motor hunting drift
- \underline{D}^p Skew symmetric matrix of error values
- \underline{D}^s Mean drift
- \underline{D}^t Torque generator drift coefficient
- \underline{D}^u Unbalance drift vector
- \underline{D}' Gyro drift vector
- \underline{D}_O Deviation of the normal
- $\underline{D}_{1x}, \underline{D}_{1y}, \underline{D}_{1z}$ Outer gimbal viscous damping factors
- $\underline{D}_{2x}, \underline{D}_{2y}, \underline{D}_{2z}$ Inner gimbal viscous damping factors
- $\delta \underline{D}^t$ Error in torque generator drift coefficient
- \underline{E}^n Skew symmetric form of $\underline{\epsilon}^n$
- \underline{F}^b Force acting on gimbal bearing
- \underline{F}^n Skew symmetric form of \underline{f}^n

t, F_2	Input functions
\mathbf{g}^1	Gravitational field vector in skew symmetric form
$\dot{r}_j(s)$	Open loop transfer functions ; $j = 1, 2, 3, 4$
\mathbf{I}	Angular momentum vector
\mathbf{I}^k	Nominal angular momentum vector
\mathbf{I}_f	Angular momentum vector with respect to frame \mathbf{f}
$\mathbf{H}_j(s)$	Feedback transfer functions ; $j = 1, 2, 3, 4$
\mathbf{H}_r	Angular momentum of rotor
\mathbf{H}_2	Angular momentum vector of inner gimbal and rotor system
\mathbf{H}_3	Angular momentum vector of rotor
\mathbf{H}_{3z}	Component of \mathbf{H}_3 in direction of \mathbf{z}_3
$\mathbf{H}_{2x}, \mathbf{H}_{2y}, \mathbf{H}_{2z}$	Components of \mathbf{H}_2 in directions $\mathbf{x}, \mathbf{y}, \mathbf{z}$
$\delta \mathbf{H}$	Error vector in angular momentum
\mathbf{I}	Identity matrix
$\hat{\mathbf{I}}$	Operator to convert vectors to skew symmetric form
\mathbf{I}_m	Imaginary part
\mathbf{I}_2	Moment of inertia of inner gimbal and rotor system
$\mathbf{I}_{2x}, \mathbf{I}_{2y}, \mathbf{I}_{2z}$	Components of \mathbf{I}_2 in directions $\mathbf{x}, \mathbf{y}, \mathbf{z}$
\mathbf{I}_{3z}	Moment of inertia of the rotor in direction of \mathbf{z}_3
\mathbf{J}	Moment of inertia
\mathbf{K}	Compliance tensor
\mathbf{K}_{ij}	Components of \mathbf{K} ; $i, j = 1, 2, 3$
$\mathbf{K}_e, \mathbf{K}_p$	Equatorial and polar constants
$\mathbf{K}_1, \mathbf{K}_2$	Gains
$\mathbf{K}_{1x}, \mathbf{K}_{1y}, \mathbf{K}_{1z}$	Outer gimbal spring rates
$\mathbf{K}_{2x}, \mathbf{K}_{2y}, \mathbf{K}_{2z}$	Inner gimbal spring rates
\mathbf{L}	Latitude angle
\mathbf{L}_c	Geocentric latitude
\mathbf{L}_o	Initial Geographic latitude
$\delta \mathbf{L}$	Latitude error
\mathbf{M}	Torque vector
\mathbf{M}^b	Gimbal bearing friction drift torque vector
\mathbf{M}^c	Command torque vector
\mathbf{M}^d	Disturbance torque vector
\mathbf{M}^t	Torque generator torque vector

$\underline{\mathbf{I}}_r$	Unwanted torques vector
$\underline{\mathbf{I}}_j$	Control torques vector
$\underline{\mathbf{I}}_m$	Error torques and applied control torques vector
$\mathbf{I}_j(s)$	Transfer functions ; $j = 1, 2, 3, 4$
$\underline{\mathbf{A}}_1$	External applied torque vector on outer gimbal
$\underline{\mathbf{A}}_2$	External applied torque vector on inner gimbal
$\mathbf{A}_{1x}, \mathbf{A}_{1y}, \mathbf{A}_{1z}$	Components of $\underline{\mathbf{M}}_1$ in directions x, y, z
$\mathbf{A}_{2x}, \mathbf{A}_{2y}, \mathbf{A}_{2z}$	Components of $\underline{\mathbf{M}}_2$ in directions x, y, z
$\mathbf{A}_{1m}, \mathbf{A}_{2m}$	Components of $\underline{\mathbf{M}}_m$ on outer and inner gimbals
\mathbf{M}_o^t	Outer gimbal torque generator torque
$(\mathbf{u})\underline{\mathbf{M}}^g$	Uncertainty torques vector
N	Universal gravitational constant
N^n	Skew symmetric form of v^n
P^k	Transformation error angles matrix
Q_j, Q_1, Q_2	Forcing functions
R	Resistance, Skew symmetric form of vector $\underline{\mathbf{r}}$
R^k	Nominal resistance
R_e	Real part
S	Actual gyro scale factor
δS	Scale factor error
S^k	Nominal gyro scale factor
S^t	Torquer scale factor
S^T	Command rate scale factor
S^{tk}	Nominal torquer scale factor
δS^t	Error in torquer scale factor
T^p, T^g	Gyro torque scale factor uncertainty matrices
T_d	Driving torque of the rotor
T_w	Windage torque
T_1, T_2	Time constants
V^n	Earth referenced velocity vector
V_N, V_E, V_D	Components of V^n in navigational frame
δV	Velocity error
X_i	Input amplitude
X_o	Output amplitude
X, Y	Coefficients of equations
Z^j	Initial system misalignment error matrix

Greek Letters

α	Input angle, error angle
$\alpha_1, \alpha_2, \alpha_3$	Misalignment angles
A	Misalignment matrix, bias
α_{ij}	Misalignment angles ; $i, j = 1, 2, 3$
δ	Uncertainty term
$\underline{\varepsilon}^n$	Attitude error vector
$\varepsilon^N, \varepsilon^E, \varepsilon^D$	Components of ε^n in navigation frame
ϕ_x, ϕ_y, ϕ_z	Euler angles
ϕ_1, ϕ_2, ϕ_3	Misalignment angles
ϕ_0	Amplitude of Euler angles
γ	Angular displacement of rotor w.r.t magnetic field
γ_{ij}	Misalignment angles term ; $i, j = 1, 2, 3$
θ_1, θ_2	Output angles of the gyro
θ_3	Rotor angular displacement in rotor frame
μ	Friction coefficient, Product of earth mass with universal Gravitational constant
\underline{v}^n	Error angles vector
v_N, v_E, v_D	Components of \underline{v}^n in navigational frame
$\underline{\rho}^k$	Vector form of P^k
ρ_x, ρ_y, ρ_z	Components of P^k in directions x, y, z
τ_x, τ_y, τ_z	Components of T^p in directions x, y, z
$\underline{\omega}$	Angular velocity
$\underline{\omega}^c$	Command rate
$\underline{\omega}^k$	Measured Angular velocity
$\underline{\omega}_f$	Angular velocity with respect to frame f
ω_n	Natural frequency
$\underline{\omega}_p$	Angular velocity with respect to frame p
ω_s	Schuler frequency
$\underline{\omega}_1$	Angular velocity of the outer gimbal
$\underline{\omega}_2$	Angular velocity of the inner gimbal
$\underline{\omega}_3$	Angular velocity of the rotor
$\underline{\omega}_{le}$	Earth Angular velocity
$\omega_x, \omega_y, \omega_z$	Components of $\underline{\omega}$ in directions x, y, z

$\omega_{px}, \omega_{py}, \omega_{pz}$ Components of $\underline{\omega}_p$ in directions x, y, z
 $\omega_{1x}, \omega_{1y}, \omega_{1z}$ Components of $\underline{\omega}_1$ in directions x, y, z
 $\omega_{2x}, \omega_{2y}, \omega_{2z}$ Components of $\underline{\omega}_2$ in directions x, y, z
 $\omega_{3x}, \omega_{3y}, \omega_{3z}$ Components of $\underline{\omega}_3$ in directions x, y, z
 $\underline{\omega}_{ij}^k$ Angular velocity of frame i w.r.t frame j projected in frame k
 $\underline{\omega}_{ip}^g$ Angular velocity of frame i w.r.t frame p projected in frame g
 $\underline{\omega}_{ip}^p$ Angular velocity of frame i w.r.t frame p projected in frame p
 $\underline{\omega}_{kj}^j$ Angular velocity of frame j w.r.t frame k projected in frame j
 $\delta \underline{\omega}$ Error in $\underline{\omega}$
 $(u)\underline{\omega}^p$ Gyro uncertainty in platform inertial angular velocity
 $(u)\omega_x, (u)\omega_y, (u)\omega_z$ Components of $(u)\underline{\omega}^p$ in directions x, y, z
 $\underline{\psi}^n$ Transformation error angles vector
 ζ Damping ratio
 $\zeta_x, \zeta_y, \zeta_z$ Components of \underline{Z}^j in directions x, y, z

Γ^k Transformation error matrix
 Λ Characteristic matrix
 Ω_{ip}^p Skew symmetric form of $\underline{\omega}_{ip}^p$
 Ω_{kj}^j Skew symmetric form of $\underline{\omega}_{kj}^j$
 Ψ^n Skew symmetric form of $\underline{\psi}^n$

(\sim) A measured value
 $(\hat{})$ A calculated value
 $(\dot{})$ Derivative w.r.t time