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EQUILIBRIUM FORMS AND STABILITY OF CHARGED DROPS

A THESIS
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Ph.D

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بسم الله الرحمن الرحيم

"سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ"

صدق الله العظيم

بسم الله الرحمن الرحيم

TO MY HUSBAND

&

MY SONS

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CONTENTS

INTRODUCTION

Equilibrium forms and stability of charged drops

Introduction.

Atomic nuclei are examples of uniformly charged drops. Although quantum effects are important in the study of atomic nuclei, the average effect of treating them as uniformly charged incompressible liquid drops (drop model of the nucleus) is quite important in the investigation of the fission of nuclei and in the study of nuclear encounters in general.

The equilibrium form of a uniformly charged drop at rest under the action of the surface tension (which keeps the positive charges from flying apart) and the Coulomb repulsion is a sphere.

A hollow sphere with spherical symmetry is a possible form of equilibrium but it can be shown that this kind of equilibrium is unstable.

It will be shown that the spherical form of a uniformly charged drop is only stable under certain conditions where a parameter x known as the fissionality parameter does not exceeds the value one. This fissionality parameter is defined as the ratio between the Coulomb energy to twice the surface energy of the spherical drop.

When the drop is not at rest, but rotates with a given constant angular momentum, then its equilibrium form will no longer be a sphere, but another complicated form. Neglecting the Coulomb energy (when the drop is uncharged) then its equilibrium form under rotation can be exactly computed (15). Increasing the angular momentum the drop finally takes an annular form and then becomes unstable under what is known as saussage deformation.

On the other hand on neglecting the surface energy of the drop and using gravitational attraction instead of Coulomb repulsion, then under rotation the drop (planets, stars, etc) takes the form of an exact oblate ellipsoid of revolution (known as Maclaurin's ellipsoids).

Increasing the angular momentum the drop can take either the form of a Maclaurin ellipsoid of revolution or as an exact triaxial ellipsoid (known as Jacobi ellipsoid).

We shall however concentrate in this thesis on the investigation of uniformly charged drops at rest. Apart from the stable equilibrium of spherical form, we shall investigate another unstable equilibrium form known as the saddle point shapes where the form is described by more than one parameter and the deformation is stable for certain variations of these parameters and unstable for other variations (investigation of the different modes of deformation). These saddle point shapes are quite important for the investigation of stability of the charged drop.

We shall follow the energy method in our investigation, either to obtain the saddle point shapes or the unstable deformation of the drop starting from the saddle point shapes.

The investigation of these unstable deformations will reveal the possibility of the fission of the drop in to several fragments.

Numerical computations are quite important in these investigation and full analytical expressions will be given which can facilitate further investigations on the subject.

Ther is however a lot of investigations carried out on the problem of stability of uniformly charged liquid drops mainly by Swiatecki, W.J and his collaborators (see for example Ref . 1,2,7,8,9,26) . In their investigations they considered the deformed drop to be given by an axially symmetric figure described in terms of 18 deformation parameters using polar coordinates γ, θ to describe the surface . Such investigations although they described exactly the saddle point shapes , they failed to describe the dynamics of the drop strating from the saddle point shapes .

In addition the computation was quite huge . We have considered the description of the axially symmetric deformed drop by using small deformations about an ellipsoid of revolution , either prolate or oblate .

This procedure was successful in describing the saddle point shapes in terms of only one big parameter (the semi-major axis a of the ellipsoid)
and a number of small deformation parameters describing the deformation about the ellipsoid .

In this way the equations of motion about the saddle point shapes could be successfully carried out and solved giving the dynamic behaveiour of the shape of the drop at successive instants of time ,

Since this procedure is new (see Ref. 4,5,6) . A full investigation on increasing the number of deformation parameters and their comparison is carefully carried out in chapter four in this thesis .

In chapter I of this thesis the posibility of fission into several fragments is investigated on using a closed form for the mutual potential energy of two ellipsoid of revolution. This was already achieved by A.Sabry and S.Shalabi (see Ref. 7 and 30). The same result is used to investigate the stability of a bubble drop.

In chapter II the saddle point shapes of a uniformly charged drop is carried out. This includes corrections in a previous work (see Ref. 4) and an extension to more deformation parameters. An investigation which is very important for establishing the present procedure with regards to other methods.

In chapter III investigations of rotating drops, using the present energy method with numerical applications were carried out.

Finally in chapter IV, as mentioned before the dynamics of the uniformly charged drops is fully investigated

In the whole thesis it was found for completeness to repeat writing same equations already appeared in our previous investigations. This was found necessary to have an understandable background to the numerous numerical applications carried out and also for any further investigation to this new procedure.

CHAPTER (I)

PRELIMINARY INVESTIGATIONS ON THE STABILITY OF CHARGED DROPS

CHAPTER I

Preliminary investigations on the stability of charged drops

In this chapter abrief investigation of the stability and the possible form of fission of a uniformly charged spherical drop is given.

We first summarize briefly the known results of limits of stability of a spherical uniformly charged drop.

In the second section the case of a hollow drop is investigated on the assumption that the inside and outside boundaries have the form of ellipsoids of revolution. The energy method is then applied to investigate the stability of the hollow drop, with the result that there always exist an unstable mode.

An approximate and easy investigation of the possible forms that an excited uniformly charged drop can have just after the fission is carried and together with some numerical results for these forms having different fissionality parameters.