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HALA ABD EL-GALIL EL-SAYED

Supervised By

Prof. Dr. Nashat F. M. Fathy
Department of Mathematics
Faculty of Science, Ain Shams University

Dr. Mahmoud K. A. Khairat
Department of Mathematics
Faculty of Science, Ain Shams University

SUBMITTED TO

Department of Mathematics
Faculty of Science
Ain Shams University

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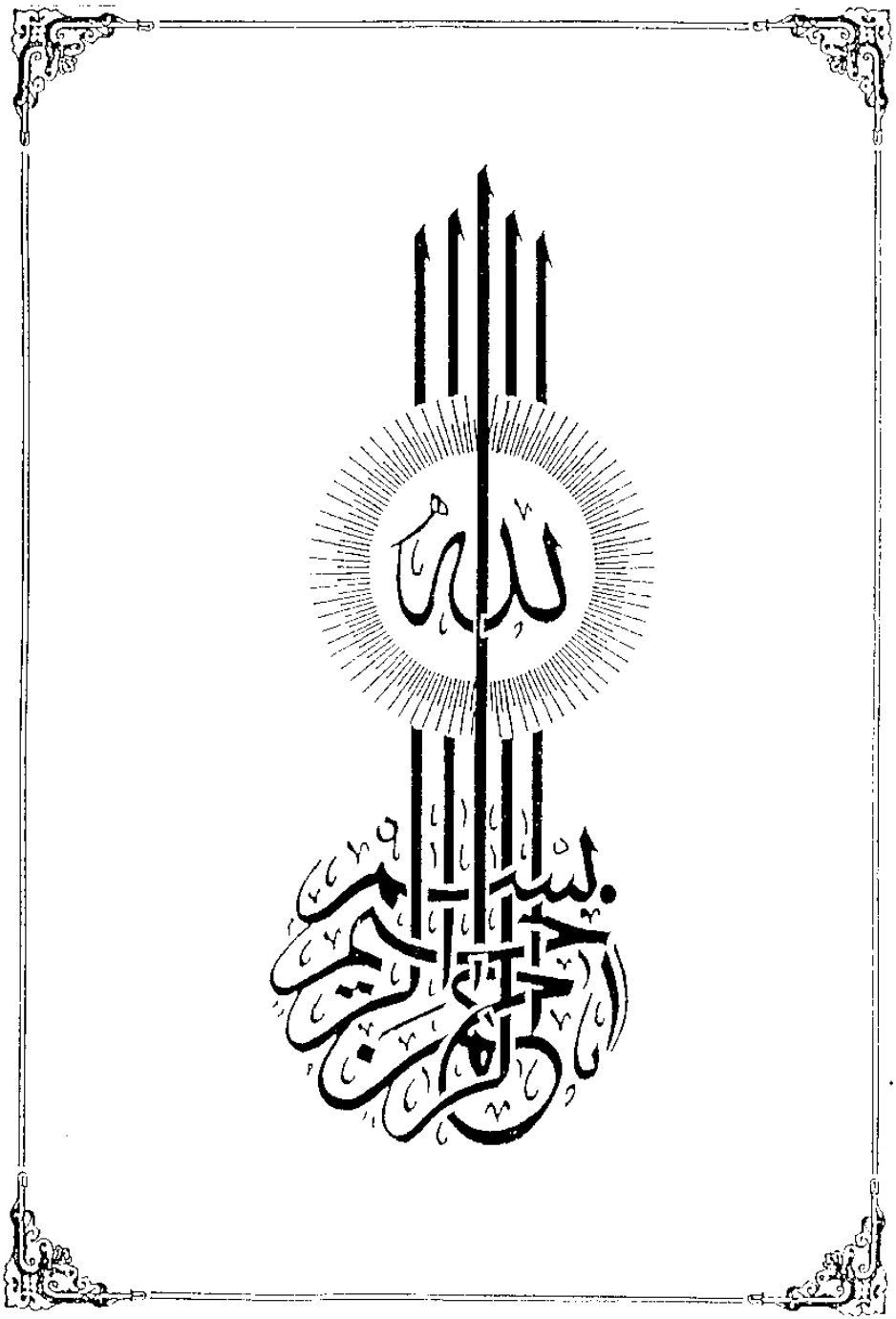

On Petri nets and their applications

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thesis advisors

Approval

Dr. Nashat Fared Mohamed
Prof. of Mathematics Dept.
Facualty of Science
Ain Shams Univesity

Dr. Mahmoud Khirat Ahmed Khirat
Prof. of Computer.Sc.
Facualty of Science
Ain Shams Univesity

(Nashat Fared)

( )

أ.د انتصارات محمد حسن الشبكي

رئيس قسم الرياضيات
كلية العلوم جامعة عين شمس

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