MHD NON-NEWTONIAN UNSTEADY FLOW IN THE PRESENCE OF TIME DEPENDENT PRESSURE GRADIENT BETWEEN TWO PARALLEL POROUS WALLS

A THESIS

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To My Family

بسم الله الرحمن الرحيم

M.SC. COURSES

STUDIED BY THE AUTHOR (1980 - 1981)

AT AIN SHAMS UNIVERSITY

FACULTY OF SCIENCE

1 - Elasticity

2 - Numerical Analysis

3 - Classical Mechanics

4 - Magnetohydrodynamics

5 - Fluid Dynamics

6 - Theory of Stability

CONTENTS

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Summary

SUMMARY

Our aim in this thesis, which consists of three chapters is to study the Magnetohydrodynamic non-Newtonian unsteady flow in the presence of time dependent pressure gradient between two parallel porous walls.

In chapter (I), we presented an introduction to the following topics:

1 - The previous studies on rheology.

2 - Rheological classification of non-Newtonian fluids.

3 - Megnetohydrodynamics.

In chapter (II), we discussed the following problem: "Magnetohydrodynamic non-Newtonian unsteady flow with variable pressure gradient between two parallel porous walls"

By applying Laplace's transform method in the case of the first approximation of the Eyring Powell model we obtained , the analytical solution for the velocity field in the form of an infinite series. To find the numerical results for the velocity field we used the computer,these results were compared with the Newtonian case and shown that the velocity field in Newtonian fluid was greater than that in the corresponding non-Newtonian fluid and for constant Reynold suction

number R_e . The difference between them decreases with increasing Hartmann number Ha , while for constant Ha, this difference increases with decreasing R_e .

Chapter (III) was devoted to solve the non-linear partial differential equation:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = - \frac{\partial P}{\partial x} + \frac{N^*}{R_e} \frac{\partial^2 u}{\partial y^2} - D^* \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \frac{Ha^2}{R_e} u.$$

By using a finite difference technique in the case of the second approximation, for the previous model, we obtained a numerical solution for the velocity field. From numerical results we had shown that :

If the first approximation parameter of non-Newtonian fluid M is constant, the velocity field increases with increasing the second approximation parameter of non-Newtonian fluid D^* , while for constant D^* , the velocity field decreases with increasing M.

In appendix (A), we clarified the finite difference technique, in appendix (B, B_1), computer programs were represented to find the numerical results for the velocity field and to solve the partial differential equation.

CHAPTER I

General Introduction

CHAPTER I
GENERAL INTRODUCTION

§(1.1) The previous studies on rheology

Boyle's law (1662), Hooke's law (1676) and Newton's viscous law (1686) which are essential for the study of many branches of physics are considered as early rheological laws, Poiseuille (1847) derived from his experiments the relationship between pressure and volumetric flow rate for the flow of blood (which we now know as non-Newtonian) through capillary tubes.

Barus (1893) also used a capillary tube, through which he extruded marine glue, and published a paper which was apparently over looked until more recent years. The important observation in his work was that the glue exhibited a time-delayed partial recovery of deformation. As far as is known, this is the first recorded direct observation of non-Newtonian behaviour and is especially important as shear elasticity has now become a major factor in the treatment of non-Newtonian flow.

There is a standard text by Van Wazer, Lyons, Kim and Colwell [1] (1963) which describes many details of viscometry. There has also recently been developed a number of more complicated devices for examining mechanical properties of fluids such as the Maxwell orthogonal rheometer and the Kepes viscometer. These instruments are relatively new, and few results have been reported from them up to the present time, common types of flow pattern used in viscometric or rheometric work are given in the texts by wilkinson [2], Lodge [3] and Walters [4] .

The theoretical investigation of non-Newtonian fluid dynamics are now on a sufficiently sound basis and also of an extent where it may be considered that it can take its place as an established subject. It can not be said that the experimental side is quite so firmly based. However, the accord between theory and experiment may be reached in non-Newtonian fluid dynamics as exists in the classical theory. This provides a common meeting ground and establishes an area of growth of engineering science.

According to the connection between steady shear properties in steady shearing motion of strongly elastic liquid and oscillatory shear properties in oscillatory shearing motion, it is proposed that three material functions are sufficient to characterise the flow of incompressible viscoelastic liquids, namely :

$$\tau_{xx} - \tau_{yy} = \sigma_1(\dot{\nu})$$

$$\tau_{yy} - \tau_{zz} = \sigma_2(\dot{\nu})$$

and $$\tau = \sigma_3(\dot{\nu})$$

Weissenberg [5 , 6] has deduced on purely theoretical grounds that $\sigma_2 = 0$ for a general class of such liquids. This appears to be at least approximately satisfied for a number of substances such as polymer solutions. It is still a matter of research to determine the range of application of this hypothesis. A through discussion of the above hypothesis has been given by Giesekus[7]in 1973.

The subject of rheometry , which may be defined as the experimental determination of the mechanical properties of matters, depends upon producing the laboratory apparatus by which we obtain the experimental data which may be used directly in the solution of an engineering problem without the entervention constitutive or rheological equation. Methods of examining the rheological properties of matter may be broadly divided into two groups according to whether the conditions at the rigid boundaries are stationary in time or variable. Variable conditions are usually obtained by imposing :

1 - A sudden stress at the solid boundaries which is subsequently held constant.

2 - A sudden strain at the solid boundaries which is subsequently held constant.

3 - A periodically varying motion at the solid boundaries.

The third case is by for the most important in examining