ON THE ASYMPTOTIC SOLUTION OF A CLASS OF LINEAR DIFFERENTIAL EQUATIONS BY THE RESIDUE METHOD

THESIS

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BY

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" ON THE ASYMPTOTIC SOLUTION OF A CLASS OF LINEAR DIFFERENTIAL EQUATIONS BY THE RESIDUE METHOD"

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COURSES

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SUMMARY

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The work presented in this thesis is devoted to the study of the spectral problem of an ordinary differential equation, together with the existence and uniqueness of the solution for a mixed problem. The main theoretical tool which is used is the Rasulov method, which can be described in general terms as follows:
Consider the mixed problem of solving a partial differential equation with initial conditions with respect to one of the independent variables and boundary conditions with respect to the other independent variable. This problem can be partitioned into two problems with a complex parameter, one of them is the boundary value problem (which we call the spectral problem) and the other is the Cauchy problem for an ordinary differential equation with respect to the time.

For the spectral problem, we establish, under certain conditions, a formula for the expansion of an arbitrary sufficiently smooth function of a real argument in a series of residues of the solution of that problem.

From the solution of the foregoing problem containing a complex parameter we construct the formal solution of the mixed problem, which can be expressed as the total residue.

The residue method was justified, for the mixed problem not containing the leading time derivative in the boundary conditions.

The thesis contains three chapters:

In the first chapter we illustrated the construction of the solution of the spectral problem. It also presents the essential definitions and theorems, that are of constant use in the sequel.

In chapter two we study the differential equation of order four with the two parameters (λ, s) where λ is a complex parameter, s is a positive integer parameter $(s \neq 0)$.

$$\frac{d^4 Y}{d y^4} - A s^4 \frac{dY}{dy} - \lambda^4 s^8 Y = h(Y),$$

with the boundary conditions

$L_1(Y) = Y(o) = 0$ $\quad$ $L_2(Y) = \grave{Y}(o) = 0$

$L_3(Y) = Y(1) = 0$ $\quad$ $L_4(Y) = \grave{Y}(1) = 0$

which is called the spectral problem.

From this study we shall obtain the asymptotic representations of the following:

The fundamental system of the particular solutions of the characteristic determinant of Green's function and its eigenvalues, the derivative of the characteristic determinant with respect to the complex parameter λ and the derivative of the numerator of Green's function at its poles.

In chapter III we shall prove the existence and uniqueness of the solution of the mixed problem by using the residue method

CHAPTER I

INTRODUCTION AND BASIC DEFINITIONS

CHAPTER [I]

"INTRODUCTION AND BASIC DEFINITIONS"

1.1 Construction Of Solution For Spectral Problem .

Let us consider the following linear differential equation

$$y^{(n)}(x,\lambda) + P_1(x,\lambda)\, y^{(n-1)}(x,\lambda) + \ldots + P_n(x,\lambda)\, y(x,\lambda) = F(x) \quad (1.1.1)$$

with the boundary conditions

$$L_\nu(y) = \sum_{k=1}^{n} \left[\alpha_{\nu k}(\lambda)\, y^{(k-1)}(a,\lambda) + \beta_{\nu k}(\lambda)\, y^{(k-1)}(b,\lambda) \right] = 0 ,\ (\nu=\overline{1,n}) \quad (1.1.2)$$

where $F(x)$, $P_k(x,\lambda)$, $(k=\overline{1,n})$ are continuous functions with respect to $x \in [a,b]$, $\alpha_{\nu k}(\lambda)$, $\beta_{\nu k}(\lambda)$ are polynomials of complex parameter λ and are independent .The functions $P_k(x,\lambda)$, $(k=\overline{1,n})$ are integral functions

$$\text{i.e} \quad P_k(x,\lambda) = \sum_{\nu=0}^{k} P_{k\nu}(x)\, \lambda^{k-\nu} \quad (1.1.3)$$

where $p_{k\nu}(x)$ are continuous functions in the interval $[a,b]$.It's clear that ,the homogenous differential equation

$$y^{(n)}(x,\lambda) + P_1(x,\lambda)\, y^{(n-1)}(x,\lambda) + \ldots\ldots\ldots\ldots + P_n(x,\lambda)\, y(x,\lambda) = 0 \quad (1.1.4)$$

can be transformed to

$$\frac{d y_n}{d \varkappa}+p_1(\varkappa,\lambda)y_n+\dots\dots\dots+p_n(\varkappa,\lambda)y_1=0 \qquad (1.1.5)$$

by the transformation

$$\left.\begin{aligned} y(\varkappa,\lambda) &= y_1(\varkappa,\lambda) \quad, \\ y'(\varkappa,\lambda) &= y_1'(\varkappa,\lambda) = y_2(\varkappa,\lambda) \quad, \\ &\;\vdots \\ y^{(k)}(\varkappa,\lambda) &= y_1^{(k)}(\varkappa,\lambda) = y_{k+1}(\varkappa,\lambda) \qquad (k=\overline{0,n-1}) . \end{aligned}\right\} \qquad (1.1.6)$$

The solution of equation *(1.1.5)* is an integral function [8]. Then equation *(1.1.4)* has a fundamental system $y_k(\varkappa,\lambda)$, $(k=\overline{1,n})$. Let the general solution of equation *(1.1.1)* be in the form

$$y(\varkappa,\lambda) = \sum_{k=1}^{n} C_k(\varkappa)\, y_k(\varkappa,\lambda) \; . \qquad (1.1.7)$$

By using the method of variation of the parameter [8] we can obtain an unknown function $C_k(\varkappa)$. Therefore, we obtain a system of n linear equations

$$\left.\begin{aligned} \sum_{k=1}^{n} C_k'(\varkappa)\, y_k(\varkappa,\lambda) &= 0 \quad, \\ \sum_{k=1}^{n} C_k'(\varkappa)\, y_k'(\varkappa,\lambda) &= 0 \quad, \\ &\;\vdots \\ \sum_{k=1}^{n} C_k'(\varkappa)\, y_k^{(n-2)}(\varkappa,\lambda) &= 0 \quad, \\ \sum_{k=1}^{n} C_k'(\varkappa)\, y_k^{(n-1)}(\varkappa,\lambda) &= F(\varkappa) . \end{aligned}\right\} \qquad (1.1.8)$$