Thesis

on

ON COLLISIONS OF Mu MESONS WITH TRITIUM MOLECULAR IONS

Presented to

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by

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SUMMARY

To study a chemical reaction from the mathematical point of view is to set up a mathematical scheme that describes the physics of the problem properly. One of these mathematical schemes is to solve the Schrodinger equation taking inot account the state that this equation describes and the restrictions imposed on this equation to fulfill the requirement of the reaction under study.

In this thesis we are devoted to the study of the scattering π^{+} meson with $({}_{1}H^{3})_{2}^{+}$ molecular ion and try to find the most suitable method for solving the associated Schrodinger equation. To do so, we follow two out of many approaches namely the algebraic expansion approach and the finite difference approach.

The out-come of these two approaches tells us that Tamm method is the most suitable of the algebraic expansion methods described, while the enhanced Numerov method is the best of the finite difference methods described. A comparative study between these two methods learns us that Tamm method is the most suitable method of all for the present study.

The following table presents a comparative study of the numerical results obtained for the last two methods.

Method	Wave number	Phase shift	Diff. cross section
Tamm	5.05	1.5212	0.3911 E-01
Enh.Numrov	5.05	1.443 E-03	0.2616 E-01

INTRODUCTION

INTRODUCTION

During the last few years the collisions between elementary particles and molecules became the take off of the link between chemical reactions and computer experiments. To study chemical reactions one has to have some knowledge of the properties of components of such reactions. One of these reactions is that of the μ-meson with the tritium molecular ion. Therefore, a brief knowledge of the history of the constitues of this reaction are worth knowing.

Nuclear Forces and Mesons:-

The nature of the forces which hold neutrons and photons together in nuclei is still not well uderstood. It is clear that the force can not be simple electric(Coulombic) attraction, because the neutron carries no charge. Gravitational forces are too weak by very many orders of magnitude to account for nuclear binding. Many experimental facts indicate that neuclear forces have a very short range, in fact a range somewhat smaller than neuclear dimensions. The type of force which is now rather generally believed to act between nucleons is a so-called exchange force, that is, a force which holds the nucleons together through a continuous exchange of some constituent particles between them.

In 1935, H.Yukawa concluded that an exchange particle about 150 times as heavy as the electron would lead to about the right magnitude and range for nuclear forces. He observed particles of approximately this mass in cosmic rays. These new particles were subsequently found to have masses equal to about 210 electron masses. They are called "mesons", from the Greek word meso, middle, because they are intermediate in mass between electrons and Protons. |1|

More recently other kinds of mesons have been discovered in cosmic rays, and several types have also been produced with high-energy accelerators.The original cosmic-ray meson with a mass of 210 electron masses, now called the μ meson, was for a long time thought to be the particle responsible for nuclear binding. But in 1946 when experiments showed that the interaction between these μ mesons and nuclei was much weaker than required by the meson theory of nuclear forces, this idea had to be abandoned. The subsequently discovered π meson (mass=275 electron masses) appears to have many of the properties required by meson theories of nuclear forces and currently the favorite candidate for the particle that is supposed to account for nuclear binding. Meson theories of nuclear forces taking into account only neutral, only charged, or both charged and neutral mesons have been developed. Yukawa suggested that mesons , if left alone, are not stable, but transform into electron. The mean life of a negative meson, before transforming to an electron, was about 10^{-8} seconds. Mesons have energies of many billions of electron vollts, they pentrate a meter of led or more and are observed to appreciable depth below water or ground.

It was found that the mesons in cosmic rays at sea level interacted very weakly with nuclei.

None of the theories is completely satisfactory; yet sufficient success has been attained to justify the hope that a really adequate meson theory of nuclear forces will eventually emerge.

The Properties of Pi-and Mu-mesons:-

The mesons which are of the type predicted by Yukawa are known as Pi-mesons or pions and denoted usually by the symbols π^+, π^- and π^0 referring to positively charged, negatively charged and neutral pions respectively. The weakly interacting mesons are called mu-mesons or muons, denoted by the symbols μ^+, μ^- .

The masses of the mesons have been measured accurately and are given in table I.

Particle	Symbol	Charge (in units of electron charge)	Mass (in units of electron mass)	Life time (sec.)	Spin quantum number
Proton	p	1	1836.6		½
Neutron	n	0	1839.0	770	½
Muons	μ^+	1	207	2.2×10^{-6}	½
	μ^-	-1	207	2.2×10^{-6}	½
Pions	π^+	1	273	2.5×10^{-8}	0
	π^0	0	265	5×10^{-15}	0
	π^{-1}	-1	273	2.5×10^{-8}	0

Table (I)

It will be noted that the neutral meson is slight less massive than its charged counter-parts |2|.

Considerable importance attaches to the determination of the intrinsic spins of the pi-and mu-mesons. It has been shown definitely that Pions have no intrinsic spin. They are therefore bosons, like photons, as might perhaps be expected because of their similar roles as intermediaties between fermions. Muons are found to be fermions possessing the same intrinsic spin, ½ quantum unit as neucleons and electrons.

The mean lifetime of the charged poins before decay into the corresponding mouns is 2.5×10^{-8} sec.. It will be seen to be about 100 times smaller than for the decay of muons into electrons or positrons.

The decay of a negative moun into an electron, or of a positive muon into a positron, cannot occur in the accordance with the conservation of momentum unless a further particle takes part. As decay process is not influenced at all by the presence of matter the possibility a second particle is emitted.
This particle must be assumed to have a very weak interaction indeed with matter as it is very difficult to observe and called neutrion which is denoted by ν.

Pion decay should be represented as

$$\pi^{\pm} \longrightarrow \mu^{\pm} + \nu \quad . \qquad (1)$$

With moun decay the simplest representation would be

$$\mu^{\pm} \longrightarrow e^{\pm} + \nu + \nu \quad . \qquad (2)$$

This is consistent with the observed result that the energy of the muon in (1) is unique but that of electron or positron in (2) is not, both decay taking place from rest. Furthermore, if the neutrino is a fermion, an even number must be produced in the decay of the type (2) since both muons and electrons are fermions. As poins are bosons,

(1) is only consistent with angular momentum conservation if an odd number of neutrinos is simultaneously produced.

The decay of the polarized μ^+ meson is accompained by asymmetry in the direction of motion of positrons. In the case of completely polarized μ^+ mesons the asymmetry in the angular distribution of positrons is at maximum. The ratio number of positrons emitted in the direction of spin of the μ^+ mesons to the number of positrons in the opposite direction, which is called the asymmetry factor c, is equal to $\frac{1}{3}$. The asymmetry disappears when the muons are depolarized.

The decay of a pion at rest, the neutrino emitted will have a definite helicity say, as indicated in fig.1

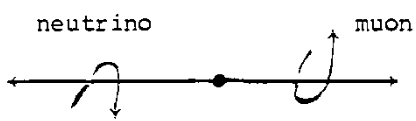

Fig. (1)

The muon must be emitted in the opposite direction to conserve momentum while, to conserve angular momentum, its spin must be equal and opposite to that of the neutrino. Hence the spin of the muon will be pointed along the direction of motion.

Tritium dating:

Tritium is a radioactive isotope of Hydrogen. The binding energy of tritium is (one proton+two neutrons) 8.5 Me.V. It is produced in the atmosphere by the action of cosmicrays. One suggestion is that it is formed by the interaction between two deuterons namely

$$D_1^2 + D_1^2 \longrightarrow {}_1H^3 + {}_1H^1$$

in which the chemical symbol is used for the nucleus, the upper suffix gives the mass number (The sum of the numbers of neutrons and protons), and the lower the atomic number (The number of electrons in the atom). The sums of each of these numbers must be the same on both sides of the reaction equations.

This reaction requires low temperature in which 4 MeV of energy is released.

We must also allow for the tritium produced in this reaction to react with a third deutron as follows:

$$_1D^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1$$

releasing a further 17.6 MeV., in which $_0n^1$ is a neutron, $_1H^1$ is a proton and $_2He^4$ alpha particle.

The decay of tritium should be represented as:

$$_1H^3 \longrightarrow {}_1He^3 + \bar{e} + \nu \ .$$

Applications of tritium dating appear to be very limited. One useful region of research is in the investigation of underground water supplies to determine whether they originate from recent rainfalls, or whether they come from large underground reserves. The half-life of tritium is 12 years and the propotion of tritium in ordinary water is very small. Radioactive isotopes get into all living plants and into an animal or human organism. Data on the content of tritium in man's body is presented in Table (II).

Isotope	Amount of isotope,g	Number of atoms	Number of disintegrations persecond
$_1H^3$	8.4×10^{-15}	1.7×10^9	3.0

Table (II)

CHAPTER I

MATHEMATICAL FORMULATION OF THE PRESENT REACTION

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1.1 Introduction:-

Of the various applications of wave mechanics to specific problems, which have made in the decade since its origin, probably the most satisfying to the chemist, are the quantitatively successful calculations regarding the structure of very simple molecules. These calculations show that we now have at hand a theory which can be confidently applied to problems of moleculare structure. They provide us with a sound concept of interactions causing atom to be held together in a stable molecule, enabling us to develop a reliable intuitive picture of the chemical bond. To a considerable extent,the contribution of wave mechanics to our understanding of the nature of the chemical bond,has consisted in the independent justification of postulates previously developed from chemical arguments, and in the removal of their indefinite character. In addition, wave mechanical arguments have led to the development of many essentially new ideas regarding the chemical bond, such as the three-electron bond, the increase in stability of molecules by resonance among several electronic structures, and the hybridization of one-electorn orbitals in bond formation.

In section 2, we are going to describe the reaction under consideration and the quantum mechanical treatment for this reaction, while in section 3, we derive the energy levels for the scattered molecular ion. Finally, in section 4, the equation for the phase shifts is to be derived.

1.2 Mechanical treatment for the present reaction:-

A chemical reaction takes place when molecules collide and a rearrangement of atoms occurs between them. This results in the production of new molecules and the liberation or absorption of energy |2| . A nuclear reaction is quite analogous, involving collisons of nuclei during which nucleons are redistributed.

There are, however, such very marked differences in the magnitudes involved that the techniques for studying the two types of reactions are wholly different.

The problem under discussion is the chemical reaction

$$({}_1H^3)_2^+ + \mu^+ \longrightarrow (\mu_1H^3)^+ + ({}_1H^3)^+ .$$

Now, the first step in treating this reaction, from the quantum mechanics point of view is to write the wave equation for the complete system either in terms of the initial state or in terms of the final state. To do so, let (r,R) denote the internal coordinates of $({}_1H^3)_2^+$ and (r',ρ_a) be the internal coordinates of $(\mu_1H^3)^+$, then the wave equation for the initial state may be written as:

$$\left[H(r,R) - \frac{\hbar^2}{2M_1} \nabla_\rho^2 + e^2\left(\frac{1}{\rho_b} + \frac{1}{\rho_a} - \frac{1}{|\underline{\rho}-\underline{r}|}\right) - E \right]\Psi = 0 \qquad (1.2.1)$$

and for the final state as:

$$\left[H'(r',\rho_a) - \frac{\hbar^2}{2M_2} \nabla_{\rho'}^2 + e^2\left(\frac{1}{R} + \frac{1}{\rho_b} - \frac{1}{r_b}\right) - E \right]\Psi = 0 \qquad (1.2.2)$$

where H and H' are the Hamiltonians for the internal motions of