THE ELASTIC STABILITY OF MULTI-BAY AND MULTI-STOREY FRAMES AND THEIR OPTIMUM PROPORTIONING UNDER DIFFERENT LOADING CONDITIONS

A THESIS SUBMITTED TO THE FACULTY OF ENGINEERING

AIN SHAMS UNIVERCITY

FOR THE DEGREE OF MASTER OF SCIENCE IN

STRUCTURAL ENGINEERING

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CAIRO, AUGUST, 1987

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بسم الله الرحمن الرحيم

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" ومن يتوكل على الله فهو حسبه وان الله بالغ أمره

قد جعل الله لكل شيء قدرا " .

صدق الله العظيم

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ACKNOWLEDGEMENTS

I wish to express my sincere appreciation and gratitude to Prof. Dr. Adel H. Salem, Dean of the Faculty of Engineering, Ain Shams University, Cairo, who supervised this work, for his guidance, constant encourgement, valuable advice and constructive criticism offered throughout the course of this study without which the effective fulfilment of this work would not have been possible.

The author wishes to express his appreciation to Dr. Mostafa K. Zidan, Assistant Professor of structural Engineering, Ain Shams University, for his kind help and his contributions in the achievement of this research.

The author also wishes to express his gratitude to the staff of the Departement of Civil Engineering of Military Technical College for their kind help offered throughout this thesis.

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NOTATION

Only the main symbols are listed here. Other symbols are defined as they appear in the text.

P = external vertical load acting on a frame;

E = Young's modulus of elasticity;

M = bending moment;

L_1 = Length of the longer column of the frame;

L_2 = Length of the short column of the frame;

L_{av} = average column length of frame;

L_b = Length of bracing diagonal;

I_1 = moment of inertia of the longer column of the frame;

I_2 = moment of inertia of the short column of the frame;

I_b = moment of inertia of frame beam;

$I_{av} = \frac{I_1+I_2}{2}$ = average column inertia ratio;

$k_1 = \frac{E\ I_1}{L_1}$ = bending stiffness of the longer column;

$k_2 = \frac{E\ I_2}{L_2}$ = bending stiffness of the short column;

$k_b = \frac{E\ I_b}{B}\cos\emptyset$ = bending stiffness of frame beam;

$k_{av} = \frac{E\ I_{av}}{L_{av}}$ = average bending stiffness of frame;

A,B,C and D = constants of integeration;

Δ = displacement of a frame;

Δ_o = displacement of a frame due to disturbing agent only;

Δ_1 = displacement of a frame due to applied load only;

Δ_2 = displacement of a frame due to applied load plus the disturbing agent;

H = Secondary force causing shear on columns produced due to sidewsay of frame;

B = Length of beam or span of frame;

$\bar{\bar{K}}$ = member stiffness matrix corresponding to the general coordinates of the system;

L = Height of colums;

P_{cr} = elastic critical load;

P_E = $\pi^2 EI/L^2$ = Euler's load of column member;

$$P_{E.av} = \frac{\pi^2 E I_{av}}{L^2_{av}}$$ = average Eulers load of column;

N = number of storeys in a frame;

c = (uL - sin uL)/(sinuL-uL cos uL) for compreced member;

s = uL(1-uL cot uL)/(2 tan(uL/2)-uL) for compreced member;

n = uL cot uL = stiffness factor for fixed end member which is in a state of no-shear sway;

n'' = uL tan uL = stiffness factor for hinged end member which is in a state of no-shear sway;

o = uL cosec uL = carry-over factor for fixed end member which is in a state of no-shear sway;

m = (2 tan (uL/2)/uL) = magnification factor for moments produced at ends of fixed member which is in a state of pure-shear sway due to axial force effect;

s'' = $s(1-c^2)$ = stiffness factor for pinned end member when sidesway is prevented;

v = secondary vertical forces produced in columns due to sidesway effect;

θ = angle of rotation;

ϕ = angle of inclination of beam , or angle of inclination of bracing diagonal;

u = $\sqrt{P/EI}$;

ρ = P/P_E = ratio of axial load to Euler's load; and

ρ_{cr} = P_{cr}/P_E = critical load parameter.

INTRODUCTION

The elastic stability of structural frameworks has been previously studied by many authors. Timoshenko (1), Bleich (2) treated single-bay single-storey hinged base rectangular frames as well as closed frames. Merchant (3,4) treated a family of symmetrical single-bay multi-storey frames with constant column sections, equal storey heights, same beam stiffnesses and loaded only at top. Salem (5, 6) studied the problem of lateral buckling of rectangular frames in which the column sections vary according to an arithmetic series. He introduced the effect of axial deformations of the frame columns due to sidesway on the elastic critical loads. Salem (7) obtained the buckling loads for non-rectangular frames permitted to sway. He considered the application of the theory of multiples in case of the elastic stability of multi-bay non-rectangular frames having equal spans. Goldberg (8,9) obtained the sway buckling loads of multi-storey Braced frames having variable column sections and loaded at each floor level (the stiffness of the frame columns varies according to a geometric series). Korashy (10) studied the lateral stability of multi-storey rectangular frames provided with different shapes of bracing diagonals and extended the analysis to include the effect of the axial deformations of columns. Zidan (11) studied the elastic buckling loads of non-rectangular multi-storey frames.

In the present thesis, several cases are studied to obtain the best proportioning of frames to satisfy maximum buckling strength with minimum frame weight. A comprehensive study of the elastic stability of non-rectangular frames having vertical columns and inclined beams is carried out in order to obtain the best proportioning of these frames. Single-bay and multi-bay non-rectangular frames as well as non-rectangular frames having intermediate horizontal girders at different heights are treated for both cases when sidesway is permitted and when it is prevented as well as for pinned base and for fixed base frames. The elastic stability of multi-bay non-rectangular frames having unequal spans is studied in order to show the applicability of the theory of multiples in case of multi-bay frames having unequal spans. Multi-bay non-rectangular frames having same roof slope are treated.

The variation of bending stiffness of columns of multi-storey rectangular frames is discussed in order