

NONLINEAR ELECTROHYDRODYNAMIC MARANGONI STABILITY

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TO :
MY PARENTS,
MY WIFE,
AND
MY CHILDREN;
SALLY AND AHMED



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SUMMARY

SUMMARY

This thesis deals with the nonlinear electrohydrodynamic Marangoni stability of both Rayleigh-Taylor and Kelvin-Helmholtz models in the presence of different electric field distributions.

This thesis consists of six chapters:

In chapter I, we explain the main aspects, the previous studies of electrohydrodynamics and their various applications. We discuss the concepts of electrohydrodynamics and stability. The equations governing the motion, the electric field and the associated boundary conditions are introduced. We explain the surface tension and adsorption and the related differential equations. The concept of Marangoni instability is explained. We present a review of both Rayleigh-Taylor and Kelvin-Helmholtz models in the last section.

In chapter II, we study the problem of electrohydrodynamic Marangoni stability in Kelvin-Helmholtz flow in the presence of a tangential electric field of an interface between two semi-infinite, dielectric, inviscid and incompressible fluids. The two fluids are assumed to be streaming in the x-direction. Also, the motion in either fluids is assumed to be irrotational. We use the method of multiple scales to expand the various perturbation quantities to yield the linear and successive nonlinear partial differential equations of the various orders. The solutions of these equations are obtained.

Chapter III is connected with the second chapter presented in this thesis, where the linear electrohydrodynamic Marangoni stability is discussed in both Rayleigh-Taylor and Kelvin-Helmholtz models. We obtained a third-order dispersion relation with real and complex coefficients for the Rayleigh-Taylor and Kelvin-Helmholtz instabilities respectively. The necessary and sufficient conditions of stability are discussed theoretically and numerically in both models. For Rayleigh-Taylor instability, we find that the critical value of the field increases with the increase of adsorption which means that the adsorption is destabilizing while the surface tension is stabilizing. For Kelvin-Helmholtz instability, the behavior is similar to that of Rayleigh-Taylor instability. Only the Kelvin-Helmholtz instability requires larger values of the field than those in the Rayleigh-Taylor instability.

In chapter IV, we study the nonlinear electrohydrodynamic Marangoni stability in both Rayleigh-Taylor and Kelvin-Helmholtz models. We get the nonlinear Schrödinger equation with complex coefficients. The surface elevation and the cutoff wavenumber in the marginal state are obtained. We discuss the stability conditions of nonlinear Schrödinger equation with complex coefficients in different cases. The numerical analysis in the marginal state is discussed for both models. For nonlinear Rayleigh-Taylor instability, we observe that the nonlinearity plays a dual role in the stability criterion regarding the effect of the electric field, the adsorption and the surface tension. For nonlinear Kelvin-Helmholtz instability, we observe

that the surface tension and the adsorption are playing a dual role, but in a manner different from that in the Rayleigh-Taylor instability.

In chapter V, we study the stability of the system at the critical point. We get the nonlinear Klein-Gordon equation with complex coefficients, while the nonlinear Klein-Gordon equation with real coefficients is obtained in the absence of interfacial adsorption. The stability conditions of the latter nonlinear Klein-Gordon equation are obtained and the stability analysis of that equation is discussed. We find that the necessary condition of stability is that either of the following conditions should be satisfied:

$$(i) \quad 0.283 < \rho < 1 \quad ; \quad \tilde{\epsilon}_1 > \tilde{\epsilon}_2 ,$$

or

$$(ii) \quad \rho < 0.283 \quad ; \quad \tilde{\epsilon}_1 < \rho \tilde{\epsilon}_2 ,$$

where $\rho = \frac{\rho_2}{\rho_1}$ is the ratio density (ρ_1 and ρ_2 are the densities of the lower and upper fluids respectively) ; $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are the lower and the upper dielectric constants respectively.

Also, we observe that the increase of the surface tension results in an increase of the critical value of the stabilizing fields while the increase of the critical wavenumber results in a decrease of the critical value of the stabilizing field.

In chapter VI, we study the problem of linear and nonlinear electrohydrodynamic Marangoni stability in both Rayleigh-Taylor and

Kelvin-Helmholtz models in the case of a normal field in the absence of surface charges on an interface between two dielectric, semi-infinite, inviscid, incompressible and immiscible fluids moving with uniform velocities in the x-direction. We use the method of multiple scales to expand the various perturbation quantities to yield the characteristic equation for the first-order and the solvability conditions for the second- and the third-orders. For the first-order, we get the dispersion relations in both Rayleigh-Taylor and Kelvin-Helmholtz models. In Rayleigh-Taylor instability, we observe that the stabilizing field increases with the increase of the wavenumber, contrary to the case of the tangential field. Also, we observe that the decrease of adsorption and the increase of surface tension are stabilizing. The previous results are the same in the Kelvin-Helmholtz instability, only the latter requires lesser values of the field than those in the Rayleigh-Taylor instability. Also this result is in contrast with the case of the tangential field.

For the higher-orders, we get nonlinear Schrödinger equation with complex coefficients and the conditions of stability for both Rayleigh-Taylor and Kelvin-Helmholtz models. For the nonlinear Rayleigh-Taylor instability, we observe that the new unstable regions in the stability diagram are decreased with the increase of adsorption, while the same regions are decreased with the decrease of surface tension. For the nonlinear Kelvin-Helmholtz instability, we observe that the newly formed stable regions allow stability for small wavenumbers, while for large wavenumbers stability is possible for

a small band of values of the electric field.

Finally, we discuss the stability at the critical point to obtain the nonlinear Klein-Gordon equation with complex and real coefficients. The stability conditions and stability analysis are discussed. We observe that the necessary condition of stability to be satisfied is that either of the following conditions should be satisfied:

$$(1) \quad \rho < 0.283 \quad ; \quad \tilde{\epsilon}_2 > \tilde{\epsilon}_1 \quad ,$$

or

$$(2) \quad 0.283 < \rho < 1 \quad ; \quad \tilde{\epsilon}_2 < \rho \tilde{\epsilon}_1 \quad .$$

CHAPTER I

INTRODUCTION

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(1.1) Electrohydrodynamics (EHD)

Electrohydrodynamics includes that part of fluid mechanics concerned with electrical force effects. It can also alternatively be considered as that of electrodynamics which is concerned with the influence of moving media on electric fields. As a matter of fact, electrohydrodynamics is a combination of these two areas. Many of the interesting problems in electrohydrodynamics involve both an effect of the fluid motion on the field, and conversely, an influence of the fields upon the motion. Although electrohydrodynamics is a relatively new subject, yet it treats an area which is not new. Its applications form the basis for major industries.

The subject deals with interaction between electric fields and free or bound charges in fluids. If an electric field interacts with multiproperty systems, surface interactions become of great importance. Waves which propagate along the interface between two fluids are called surface waves. In case a deformation takes place in the interface between two fluids in presence of an electric field, there are additional energy storages. When the electromagnetic energy is stored predominately in the electric field, the interaction is called an electrohydrodynamic interaction.

Electrohydrodynamic instability at the interface between two fluids has received attention in the context of many and varied physical configurations. Most often considered are situations where the fluid is much more conducting and has an electrical relaxation time short compared with the electromechanical dynamics of interest,